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13. ABSTRACT (Maximum 200 words)

Research in the following areas is reported: (A) Scattering of high-frequency sound by elastic objects in water--Experiments and theory are examined mostly for problems not describable by thin shell theory including: backwards-wave high-frequency enhancement of backscattering by shells; coincidence frequency enhancement for chirped bursts and impulses; a novel impulse generator and the effect of the mass-per-area of the shell on the specular return; and the application of time-frequency analysis. Some leaky wave scattering properties are examined including the Fresnel width of the coupling region, a convolution formulation, and a variable phase coupling coefficient. Retro-reflective backscattering of sound due to Rayleigh waves on objects with corners is demonstrated. (B) Radiation mechanisms for fluid-loaded plates--A flexural wavepacket is launched to propagate down a plate that has only its bottom half submerged. When the packet crosses the free surface of the water, there is a burst of acoustic radiation into the water due to the jump in fluid loading. This transition radiation is measured and modeled. (C) Caustic wavefields and diffraction catastrophes--Echo fluctuations associated with caustics in wavefields produced by reflection from rough surfaces and optical observations of $E_6$ diffraction catastrophes. (D) Interaction of sound with sound mediated by a suspension.

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# SCATTERING AND RADIATION OF HIGH FREQUENCY SOUND IN WATER BY ELASTIC OBJECTS, PARTICLE SUSPENSIONS, AND CURVED SURFACES


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July 1994

**Annual Summary Report for Grant N00014-92-J-1600**

1 July 1992 - 31 May 1994

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Prepared for: **Office of Naval Research**
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**800 North Quincy Street**
**Arlington, VA 22217-5660**

## ABSTRACT

Research in the following areas is reported in this Annual Summary Report:

A. **Scattering of high-frequency sound by elastic objects in water**--Experiments and theory are examined mostly for problems not describable by thin shell theory including: backwards-wave high-frequency enhancement of backscattering by shells; coincidence frequency enhancement for chirped bursts and impulses; a novel impulse generator and the effect of the mass-per-area of the shell on the specular return; and the application of time-frequency analysis. Some leaky wave scattering properties are examined including the Fresnel width of the coupling region, a convolution formulation, and a variable phase coupling coefficient. Retro-reflective backscattering of sound due to Rayleigh waves on objects with corners is demonstrated.

B. **Radiation mechanisms for fluid-loaded plates**--A flexural wavepacket is launched to propagate down a plate that has only its bottom half submerged. When the packet crosses the free surface of the water, there is a burst of acoustic radiation into the water due to the jump in fluid loading. This transition radiation is measured and modeled.

C. **Caustic wavefields and diffraction catastrophes**--The principal effort concerns the characterization of echo fluctuations associated with caustics in the wavefield produced by reflection from rough surfaces. Supporting optical experiments concern the observation of $E_6$ diffraction catastrophes.

D. **Interaction of sound with sound mediated by a suspension of particles**--Alternative measurement configurations are being explored to make it easier to relate properties of the suspension to acoustic signatures.

TABLE OF CONTENTS

# I. INTRODUCTION AND ORGANIZATION OF THIS REPORT

This Annual Summary Report is organized as follows. Section II lists the personnel involved in the research. Section III gives for the period covered a bibliography of publications, reports, dissertations, and communications resulting from the sponsored research. The items are listed to facilitate easy reference in the main body of the report. Section IV summarizes selected research results grouped according to the four principal task areas originally proposed: (A) scattering of high-frequency sound by elastic objects in water, (B) radiation mechanisms for fluid loaded membranes and plates, (C) caustic wavefields and diffraction catastrophes, and (D) the interaction of sound with sound mediated by a suspension of particles. Summarized in the final subsection is (E) supplemental research and scholarship. Section V, which is Appendix A, gives a paper that has been submitted for publication which describes aspects of the research.

# II. PERSONNEL

The following persons participated in the research.

## Graduate students

1. D. H. Hughes: Completed Ph.D. degree in December 1992 on theoretical and computational problems pertaining to scattering by elastic shells. [Hughes is currently an ONR Postdoctoral Fellow at the Physical Acoustics Branch of NRL (Washington, D.C.).]

2. T. J. Matula: Completed Ph.D. degree in December 1993 pertaining to the radiation and scattering by flexural waves on plates. (Matula is currently in a postdoctoral position at APL, Univ. of Washington.)

3. G. Kaduchak: Ph.D. candidate working on scattering by elastic shells.

4. K. Gipson: Ph.D. candidate working on scattering related problems. (Position facilitated by AASERT augmentation.)

5. C. Kwiatkowski: Ph.D. candidate working on ultrasonic four-wave mixing mediated by particle suspensions and related aspects of nonlinear acoustics. He also completed an M.S. degree project on a novel transient source with support from this project. (Position facilitated by AASERT augmentation.)

6. D. Garvey: Ph.D. candidate who worked on radiation mechanisms for fluid loaded elastic objects prior to changing to a different research program at Washington State University.

**Other personnel**

1. P. L. Marston: principal investigator

III. BIBLIOGRAPHY OF PUBLICATIONS AND REPORTS FOR GRANT N00014-92-J-1600

Research publications, pending publications, dissertations, technical and internal reports, reports of related previous work, and abstracted oral presentations are listed below according to category. Also listed for reference in the body of the report is a letter code. Thus, for example, J1 refers to the first journal publication. Each list is roughly chronological. Items supported in part by other sources are so indicated. (Publications supported by the expired grant N00014-89-J-3088 which were published after the final report was issued in 1992 are listed here and are indicated by **.)

**Code J: Refereed Journal Publications.** Items that have been accepted for publication are so indicated while items submitted and under review are listed only as "submitted."

J1. P. L. Marston and N. H. Sun, "Resonance and interference scattering near the coincidence frequency of a thin spherical shell: an approximate ray synthesis." J. Acoust. Soc. Am. **92**, 3315-3319 (1992).**

J2. G. Kaduchak and P. L. Marston, "Observation of the midfrequency enhancement of tone bursts backscattered by a thin spherical shell in water near the coincidence frequency," J. Acoust. Soc. Am. **93**, 224-230 (1993).**

J3. T. J. Matula and P. L. Marston, "Electromagnetic acoustic wave transducer for the generation of acoustic evanescent waves on membranes and an optical wave-number selective detector," J. Acoust. Soc. Am. **93**, 2221-2227 (1993).**

J4. T. J. Matula and P. L. Marston, "Diffraction of evanescent wave tone bursts on a membrane in air," J. Acoust. Soc. Am. **93**, 1192-1195 (1993).**

J5. G. Kaduchak and P. L. Marston, "Backscattering of chirped bursts by a thin spherical shell near the coincidence frequency." J. Acoust. Soc. Am. **93**, 2700-2706 (1993).

J6. D. H. Hughes and P. L. Marston, "Local temporal variance of Wigner's distribution function as a spectroscopic observable: Lamb wave resonances of a spherical shell," J. Acoust. Soc. Am. **94**, 499-505 (1993).

J7. J. S. Stroud and P. L. Marston, "Optical detection of transient bubble oscillations associated with the underwater noise of rain," J. Acoust. Soc. Am. **94**, 2788-2792 (1993). (Partially supported also by N00014-91-J-1374.)

J8. C. K. Frederickson and P. L. Marston, "Travel time surface of a transverse cusp caustic produced by reflection of acoustical transients from a curved metal surface in water," J. Acoust. Soc. Am. **95**, 650-660 (1994).**

J9. G. Kaduchak, P. L. Marston, and H. J. Simpson, "$E_6$ diffraction catastrophe of the primary rainbow of oblate water drops: Observation with white light and laser illumination," Applied Optics (accepted for publication).

J10. G. Kaduchak and P. L. Marston, "Hyperbolic umbilic and $E_6$ diffraction catastrophes associated with the secondary rainbow of oblate water drops: Observations with laser illumination," Applied Optics (accepted for publication).

J11. P. L. Marston and G. Kaduchak, "Generalized rainbows and unfolded glories of oblate drops: Organization of multiple internal reflections and extension of cusps into Alexander's dark band," Applied Optics (accepted for publication).

J12. P. L. Marston, "Leaky waves on curved scatterers: I. Fresnel width of coupling regions and elliptical Fresnel patches," J. Acoust. Soc. Am. (accepted for publication).

J13. K. L. Williams, J. S. Stroud, and P. L. Marston, "High frequency forward scattering from Gaussian spectrum, pressure release, corrugated surfaces. I: Catastrophe theory modeling," accepted for publication in J. Acoust. Soc. Am. (K. L. Williams supported by other ONR resources; J. S. Stroud supported by subcontract 721569 from University of Washington.)

J14. G. Kaduchak, D. H. Hughes, and P. L. Marston, "Enhancement of the backscattering of high-frequency tone bursts by thin spherical shells associated with a backwards wave: Observations and ray approximation," J. Acoust. Soc. Am. (accepted for publication).

J15. G. Kaduchak and P. L. Marston, "Traveling-wave decomposition of surface displacements associated with scattering by a cylindrical shell: Numerical evaluation displaying guided forward and backward wave properties," submitted to J. Acoust. Soc. Am.

J16. P. L. Marston and N. H. Sun, "Backscattering near the coincidence frequency of a thin cylindrical shell: an approximate ray synthesis," submitted to J. Acoust. Soc. Am.

J17. P. L. Marston, "Variable phase coupling coefficient for leaky waves on spheres and cylinders from resonance scattering theory," Wave Motion (accepted for publication).

J18. T. J. Matula and P. L. Marston, "Energy branching of a subsonic flexural wave on a plate at an air-water interface. I: Observation of the wave field near the interface and near the plate," submitted to J. Acoust. Soc. Am.

J19. G. Kaduchak, C. S. Kwiatkowski, and P. L. Marston, "Measurement and interpretation of the impulse response for backscattering by a thin spherical shell using a broad-bandwidth source that is nearly acoustically transparent," submitted to J. Acoust. Soc. Am.

**Code B: Chapters in Books.**

B1. P. L. Marston (editor), *Selected Papers on Geometrical Aspects of Scattering* (SPIE Optical Engineering Press, Bellingham, WA, 1994) 716 pages + xix.

B2. S. M. Bäumer, D. L. Kingsbury, and P. L. Marston, translators and editors of "Das elektromagnetische Feld um einen Zylinder und die Theorie des Regenbogens," by P.

Debye, *Physikalische Zeitschrift*, Vol. 9(22), pp. 775-778 (1908) [Translation published in pp. 198-204 of book noted above].

B3. J. S. Stroud and P. L. Marston, "Transient Bubble Oscillations Associated with the Underwater Noise of Rain Detected Optically and Some Properties of Light Scattered by Bubbles," accepted for publication in *Dynamics and Interface Phenomena* (Kluwer Publisher). Peer reviewed publication based on 1993 IUTAM meeting presentation (by invitation only).

**Code T: Thesis, Dissertations, Technical Reports, or Other Reports.**

T1. D. H. Hughes, "Backscattering of Sound by Spherical Shells in Water," Ph.D. dissertation, Department of Physics, Washington State University (1992), 240 pages [abstract published in J. Acoust. Soc. Am. **94**, 1168 (1993)].

T2. T. J. Matula, "Generation, Diffraction, and Radiation of Subsonic Flexural Waves on Membranes and Plates: Observations of Structural and Acoustical Wave Fields," Ph.D. dissertation, Department of Physics, Washington State University (1993) 201 pages [abstract published in J. Acoust. Soc. Am. **95**, 3672 (1994)].

T3. P. L. Marston, "Scattering of Radiation by Shells and Nonlinear Acoustics of Particle Suspensions," (Final Report for N00014 -89-J-3088 issued October 1992) DTIC Accession No. AD-A257257, 67 pages.

T4. P. L. Marston, "Ray Methods for Acoustic Scattering, Optics of Bubbles, Diffraction Catastrophes, and Nonlinear Acoustics," (Final Report for N00014-85-C-0141 issued November 1992) DTIC Accession No. AD-A258938, 47 pages.

**Code P: Papers In Conference Proceedings.**

P1. H. J. Simpson and P. L. Marston, "Ultrasonic four-wave mixing mediated by a suspension," in *Advances in Nonlinear Acoustics*, Proceedings of the 13th International Symposium on Nonlinear Acoustics, edited by H. Hobaek (World Scientific, Singapore, 1993) pp. 644-649.

P2. G. Kaduchak, P. L. Marston, and H. J. Simpson, "Observation of the $E_6$ diffraction catastrophe associated with the primary rainbow of oblate drops," in *Light and Color*

*in the Open Air Technical Digest, 1993,* Vol. 13 (Optical Society of America, Washington, D.C., 1993) pp. 5-7.

P3. P. L. Marston and G. Kaduchak, "Secondary and higher-order generalized rainbows and unfolded glories of oblate drips: analysis and laboratory observations," in *Light and Color in the Open Air Technical Digest, 1993,* Vol. 13 (Optical society of America, Washington, D.C., 1993) pp. 12-15.

P4. P. L. Marston, D. H. Hughes, G. Kaduchak, and T. J. Matula, "High-frequency radiation and scattering processes for shells and plates in water: Backwards waves, coincidence enhancements, and transition radiation," submitted to: Third International Congress on Air- and Structure-Borne Sound and Vibration. (Manuscript reproduced in this report in Appendix A.)

**Code I: Internal Reports.** These include various reports on file that were not widely distributed. Copies will be available for a limited time from the principal investigator. Some of these have been superseded by other manuscripts that were submitted for publication.

I1. G. Kaduchak, "Backscattering of chirped bursts by a thin spherical shell near the coincidence frequency," October 1992. Note: This paper (and associated presentation) by Kaduchak resulted in the Second Place Award for a student paper on Structural Acoustics and Vibration, New Orleans Meeting of the Acoustical Society of America, November 1992.

I2. C. S. Kwiatkowski, "Broadband impulse transducer for the measurement of backscattering objects in water," M.S. degree project report, Physics Dept., Washington State University (1993).

**Code M: Oral Presentations at Professional Meetings.** Presentations at meetings having published proceedings are listed in code P (above). Unless otherwise noted presentations listed below were at national meetings of the Acoustical Society of America and an abstract was published. Various invited research presentations at government research centers or universities are not listed.

M1. G. Kaduchak, "Backscattering of chirped bursts by a thin spherical shell near the coincidence frequency," J. Acoust. Soc. Am. **92**, 2462 (1992). (See item I1.)

M2. G. Kaduchak and P. L. Marston, "$E_6$ diffraction catastrophe in light scattered near the rainbow region of an acoustically levitated spheroidal water drop," J. Acoust Soc. Am. **92**, 2474 (1992).

M3. P. L. Marston and N. H. Sun, "Liquid-filled spherical reflectors: Analysis of glory ray amplitudes," J. Acoust. Soc. Am. **92**, 2472 (1992).

M4. P. L. Marston and N. H. Sun, "Liquid-filled spherical reflectors: the exceptional case of refractive index approaching two," J. Acoust. Soc. Am. **92**, 2472 (1992).

M5. P. L. Marston, "Classical sound waves as a coherent superposition of phonons," J. Acoust. Soc. Am. **93**, 2312 (1993).

M6. H. J. Simpson and P. L. Marston, "Ultrasonic four-wave mixing mediated by a suspension of microspheres in water: Comparison between two scattering theories," J. Acoust. Soc. Am. **93**, 2384 (1993).

M7. P. L. Marston, "Fresnel width of the coupling regions of generalized leaky Lamb waves and Fermat's principle," J. Acoust. Soc. Am. **93**, 2411 (1993).

M8. D. H. Hughes and P. L. Marston, "Time-frequency spectrograms of impulse scattering by shells: Quantitative comparisons with ray theory of Lamb wave contributions," J. Acoust. Soc. Am. **94**, 1823 (1993).

M9. C. S. Kwiatkowski, G. Kaduchak, and P. L. Marston, "Broadband impulse transducer for measurement of backscattering by objects in water," J. Acoust. Soc. Am. **94**, 1831 (1993).

M10. G. Kaduchak, C. S. Kwiatkowski, and P. L. Marston, "Impulse response for backscattering by a thin spherical shell: Measurement and wave interpretation," J. Acoust. Soc. Am. **94**, 1877 (1993).

M11. G. Kaduchak, T. J. Matula, and P. L. Marston, "Traveling wave decomposition of surface displacements on a cylindrical shell: Numerical evaluation displaying guided wave properties," J. Acoust. Soc. Am. **94**, 1861 (1993).

M12. T. J. Matula and P. L. Marston, "Energy branching of a subsonic flexural wave on a plate at an air-water interface: Transition radiation and the acoustic wave field in water," J. Acoust. Soc. Am. **94**, 1877 (1993).

M13. G. Kaduchak and P. L. Marston, "Observation of the prompt high-frequency enhancement of tone bursts backscattered by a thin spherical shell near the first longitudinal resonance," J. Acoust. Soc. Am. **94**, 1877 (1993).

M14. P. L. Marston, "Convolution formulation of leaky wave contributions to scattering by plates and by cylinders and shells of variable curvature," J. Acoust. Soc. Am. **94**, 1861 (1993).

M15. J. S. Stroud, P. L. Marston, and K. L. Williams, "High-frequency forward scattering from Gaussian spectrum, pressure release, corrugated surfaces: Experiment and comparison with catastrophe theory," J. Acoust. Soc. Am. **94**, 1891 (1993).

## IV. RESEARCH PROJECTS

This discussion summarizes the research results in the context of previous work by others and by Marston and associates. The reader should see the various publications and reports listed in Section III for theoretical and technical details as well as the Appendix to this report.

### A. Scattering of high-frequency sound by elastic objects in water.

The emphasis of this research program has been on high-frequency scattering for situations where the elastic contributions to backscattering are large. For situations where the scatterer is an elastic shell, various excited elastic waves of interest are not desirable by theories for scattering by thin shells being developed by other researchers. Thus, for example, it is usually necessary to rely on dispersion relations based on the full equations of elasticity with fluid loading (instead of the much simpler equations of a fluid-loaded "thin shell"). This restriction is a consequence of the scattering processes of interest occurring at high frequencies where the assumptions of "thin shell theory" break down irrespective of the shell thickness. Some of the results are also applicable to leaky Rayleigh waves and to aspects of ultrasonic nondestructive testing.

From January through mid April of 1993 Marston was a visiting scientist with the Advanced Sonar Division of the Applied Research Laboratories of the University of Texas at Austin and was supported by the ARL:UT Independent Research and Development Program. During this period he was able to show how some of the methods developed with the support of this grant have application to scattering from some targets on interest at high-frequencies used in high-resolution sonar systems.

Some specific accomplishments and areas of progress will now be summarized.

## 1. Backscattering of chirped tone bursts near the coincidence frequency of a thin shell: experiments and analysis.

One objective of these experiments was to determine the effects of sweeping an incident tone burst through the region of the coincidence frequency for an empty thin spherical shell. Previously a large enhancement had been observed and analyzed for tone bursts of fixed carrier frequency and short duration [Zhang, Sun, and Marston, J. Acoust. Soc. Am. **91**, 1862-1874 (1992); Kaduchak and Marston, J. Acoust. Soc. Am. **93**, 224-230 (1993)]. For metallic objects, the frequency of the enhancement is typically close to the condition $kh \approx 1$ where h is the shell thickness which may be estimated from knowledge of the enhancement. The new experiments and analysis show that the enhancement is not destroyed by sweeping through the coincidence region. For the greatest enhancement, the incident signal should be chirped from high-to-low at a rate where the scatterer acts like a matched filter to give maximum compression of the scattered burst. These results are described in a paper published in the *Journal of the Acoustical Society of America*[J2] and are summarized here in a section of Appendix A.

## 2. Observations of the backwards-wave high-frequency enhancement of backscattering by a shell.

Also, summarized in a section of Appendix A is research into a large enhancement of the backscattering that was observed and modeled. As illustrated by Fig. 3 of Appendix A the rays of interest do not propagate around the shadow side of the scatterer so that the detailed shape of the shadow side of the scatterer is irrelevant. D. H. Hughes contributed to the theory for the process in his 1992 Ph.D. dissertation.[T1] A detailed discussion of Kaduchak's experiments along with the theory has been accepted for publication in the *Journal of the Acoustical Society of America.*[J14] **Figure 1** reproduces the abstract of this manuscript which serves to summarize the results. **Figure 2** shows a comparison of the measured and predicted scattering enhancements (relative to reflection by a rigid sphere of the same size). The comparison shows good agreement between the experiment and the

Accepted for publication in J. Acoust. Soc. Am.



**Enhancement of the backscattering of high-frequency tone bursts by thin spherical shells associated with a backwards wave: Observations and ray approximation**

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## ABSTRACT

A prominent feature predicted for the backscattering of tone bursts by thin spherical shells is an enhancement of a guided wave contribution near the first longitudinal resonance. This has been explained with a backward ray model of a leaky Lamb wave where energy is leaked off without having circumnavigated the far side of the shell [P. L. Marston et al., J. Acoust. Soc. Am. **90**, 2341 (1991); D. H. Hughes, Ph.D. thesis, Washington State University (1992)]. The relevant $s_{2b}$ Lamb wave has opposing group and phase velocities giving rise to prompt radiation following the direct specular echo. The present research gives a comparison between a ray theory approximation and experiments in which tone bursts having carrier frequencies in the range $585 < ka < 630$ were incident on an empty stainless steel spherical shell of radius $a = 12.7$ cm in water. The sphere's thickness to radius ratio is approximately 0.02. Measurements of the superposition of the specular reflection and $s_{2b}$ Lamb wave contribution agree with the predicted amplitudes and are nearly a factor of 5.5 larger than the reflection from a rigid sphere of the same size. The calculated properties of the $s_{2b}$ wave are compared with calculations for a shell in vacuum.

PACS Numbers: 43.20.Fn, 43.30.Gv, 43.35.Cg

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Figure 1

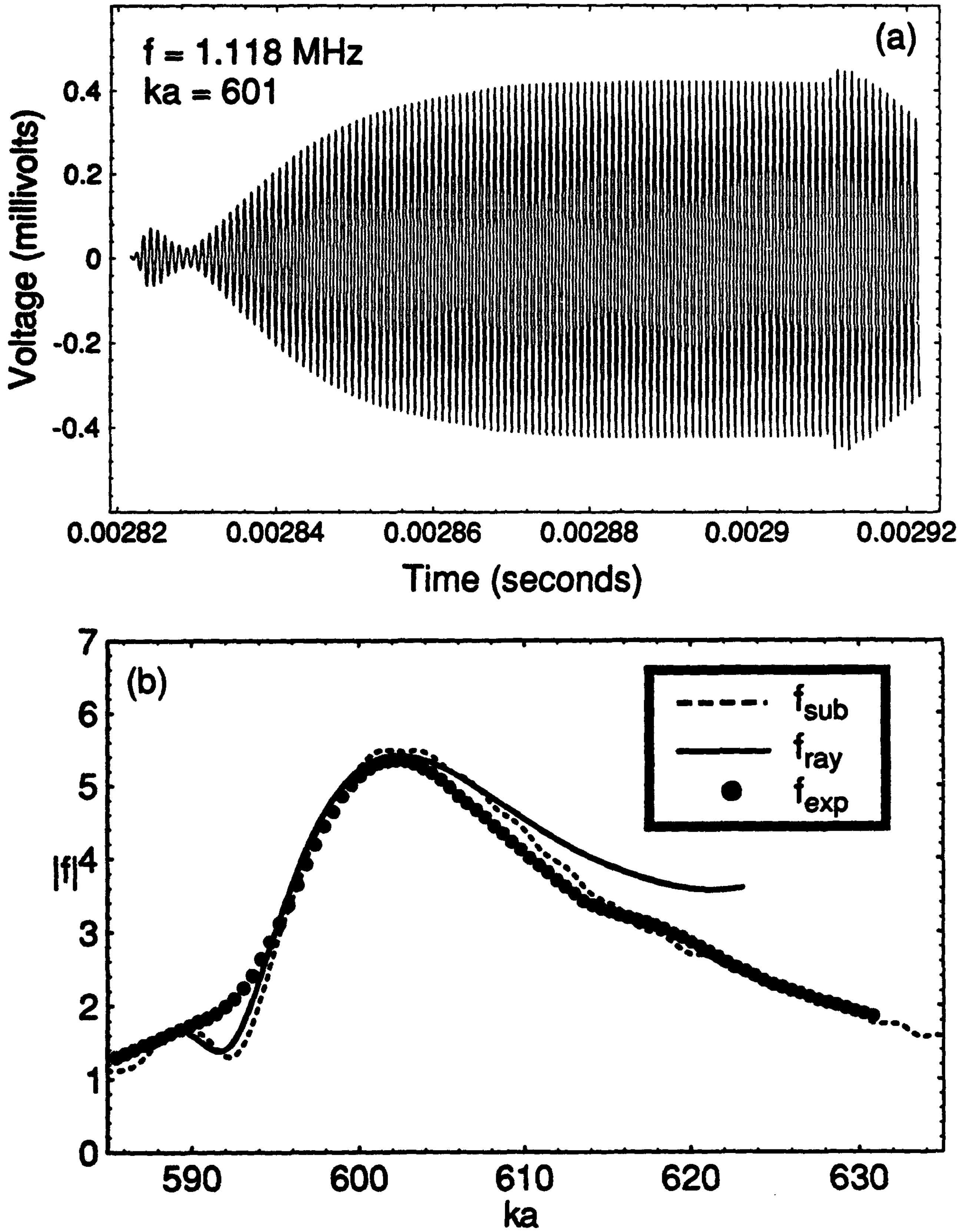


Figure 2 (a) Experimental record, (b) Comparison with theory computational result based on a background subtraction. The ray approximation is also in good agreement at the highest frequency side where the enhancement is weaker. In related work the actual surface displacements on a cylindrical shell were computed with an incident sound wave burst at a frequency in the region of the backwards wave. The displacements manifest a backwards wave having group and phase velocities in agreement with theory. These results were presented[M11] and submitted for publication.[J15]

### 3. Time-frequency analysis of scattering by thin shells based on smooth Wigner distribution function and related derived quantities.

Hughes explored in his 1992 Ph.D. dissertation[T1] the application of the smooth Wigner distribution function (SWDF) to the information content of the impulse response of a thin shell. One aspect of this research, which was published in the *Journal of the Acoustical Society of America*[J6], shows how a derived quantity, the local temporal variance (or temporal spread) of the raw WDF, has potential application as a spectroscopic observable in scattering measurements. The analysis makes quantitative the idea that the impulse response decays more slowly for frequency bands close to a resonance. In related work, the decay rate for the amplitude of wave packets in a frequency band was directly measured from a numerically evaluated SWDF of the impulse response for a 2.5% thick stainless steel shell in water. **Figure 3(a)** shows the SWDF magnitude as a function of the normalized time T = tc/a for a broad frequency band near $ka = 150$ where $a$ is the radius of the sphere. The specular reflection occurs at T = 1 and is not shown. The regular sequence of packets shown in Fig. 3(a) corresponds to successive circumnavigations of the $s_0$ leaky Lamb wave. The exponential decay rate of these packets was measured as a function of $x = ka$ by constructing slices with different times like Fig. 3(a) but for different values of x. The envelope of these packets was proportional to the function

$\exp[-T\gamma_{s0}/2]$ where

$\gamma_{s0}$ = decay rate of $s_0$ leaky Lamb wave.

The $\gamma_{s0}$ determined from slices through the SWDF are shown by the points in **Fig. 3(b)**. Ray theory based on the Watson transformation formalism predicts that[T1]

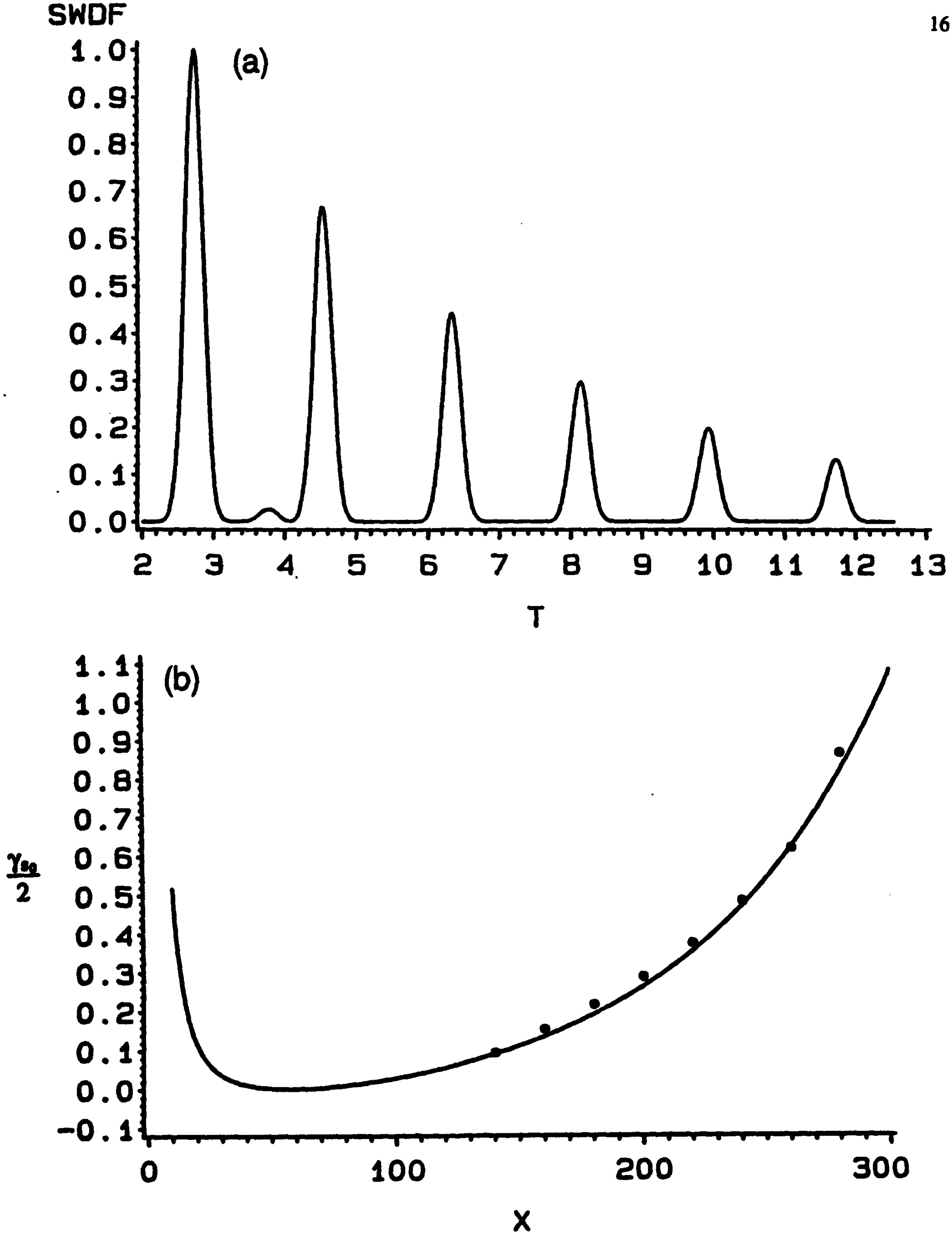


Figure 3. (a) Slice through a SWDF of the impulse response of a thin shell. (b) Comparison of decay rate from slices like in (a) with ray theory for various x = ka.

$\gamma_l = 2\beta_l c_{gl}/c$ ,

$\beta_l$ = damping of $l$th class of leaky wave in Np /radian

$c_{gl}$ = group velocity.

The curve in Fig. 3(b) is the ray theory result (base on $\beta_l$ and $c_{gl}$ from elasticity theory for $l = s_0$). There is good agreement with the result directly from the smooth Wigner distribution function.

## 4. Novel source of plane wave impulses and application to measuring the impulse response for backscattering by thin shells.

A new transducer configuration was developed with the support of this grant for the measurement of the response of a target to a plane wave impulse. The source consisted of a large sheet of PVDF piezoelectric polymer. In response to a step voltage input, the radiated pressure signature approximates a pressure impulse for spectral components below about 1 MHz. (The detailed width of the unipolar pulse depends on the sheet dimensions and the power source.) The source was originally tested as a 15 cm × 15 cm sheet and subsequently larger sheets were developed for scattering experiments (45 cm × 45 cm and 90 cm × 90 cm). The work has been presented [M9,M10] and submitted for publication.[J19] **Figure 4** gives the abstract of the manuscript. **Figure 5(a)** shows a record of the pressure impulse radiated by the 45 × 45 cm sheet while **Fig. 5(b)** shows the configuration of the backscattering experiment. **Figure 6(a)** records the response of a shell with a radius $a$ = 38.1 mm and a thickness h such that $h/a$ = 0.023. (Care was taken to subtract off a hydrophone/background signal direct from the source.) **Figure 6(b)** shows the calculated impulse response based on the theoretical form function weighted by an approximate model of the frequency response of the source-hydrophone system. Some of the features in common of the measured and calculated impulse responses are the following: (a) an initial specular response having a bipolar character determined by the mass-per-area of the shell (see Sec. 5, below); (b) a sequence of high-frequency wave packets having a frequency ≈ 300 kHz; (c) those packets are superposed on a low-frequency (10 kHz) oscillation, (d) a periodic bipolar transient of intermediate frequency. Features (b), (c), and (d) are found to be associated with the following classes of waves that circumnavigate the shell: (b) the $a_{0-}$ coincidence frequency wavepackets found in previous computations[J5]; (c) a low-frequency radiation enhancement of the $a_{0-}$ wave present for spheres; and (d) wavepackets of the $s_0$ leaky Lamb wave. A remarkable feature of the experiment is that a

# Measurement and interpretation of the impulse response for backscattering by a thin spherical shell using a broad-bandwidth source that is nearly acoustically transparent

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## Abstract

A broad-bandwidth sheet source was developed to produce a pressure impulse with a planar wavefront containing a wide range of frequency components . The source consisted of a PVDF sheet with water in contact with both sides. The PVDF was driven by a step voltage. This source is nearly acoustically transparent and was used for backscattering from an empty stainless steel spherical shell where prominent features in the shell's calculated impulse response are observed over a wide frequency interval. The shell was placed in the near field of the source where it experienced an impulsive pressure pulse followed much later by contributions from the finite source size. A wide bandwidth hydrophone was placed in the far field of the scatterer on the opposite side of the source. Time records reveal a Gaussian wave packet associated with the excitation of the subsonic $a_{0-}$ wave responsible for a large backscattering enhancement near the coincidence frequency. Superposed on the same records are large contributions from the low frequency excitation of the $a_{0-}$ wave and the $s_0$ wave [G. Kaduchak and P. L. Marston, J. Acoust. Soc. Am. 93, 2700-2706 (1993)]. A bipolar feature of the initial response was observed and was found to be associated with the finite inertia of the shell. The shell used in the experiment has a thickness to radius ratio of 2.3% for which these scattering phenomena occur between 8 and 450 kHz.

**PACS Numbers:** 43.20.Px, 43.20.Fn, 43.40.Ey, 43.88.Fx

Figure 4

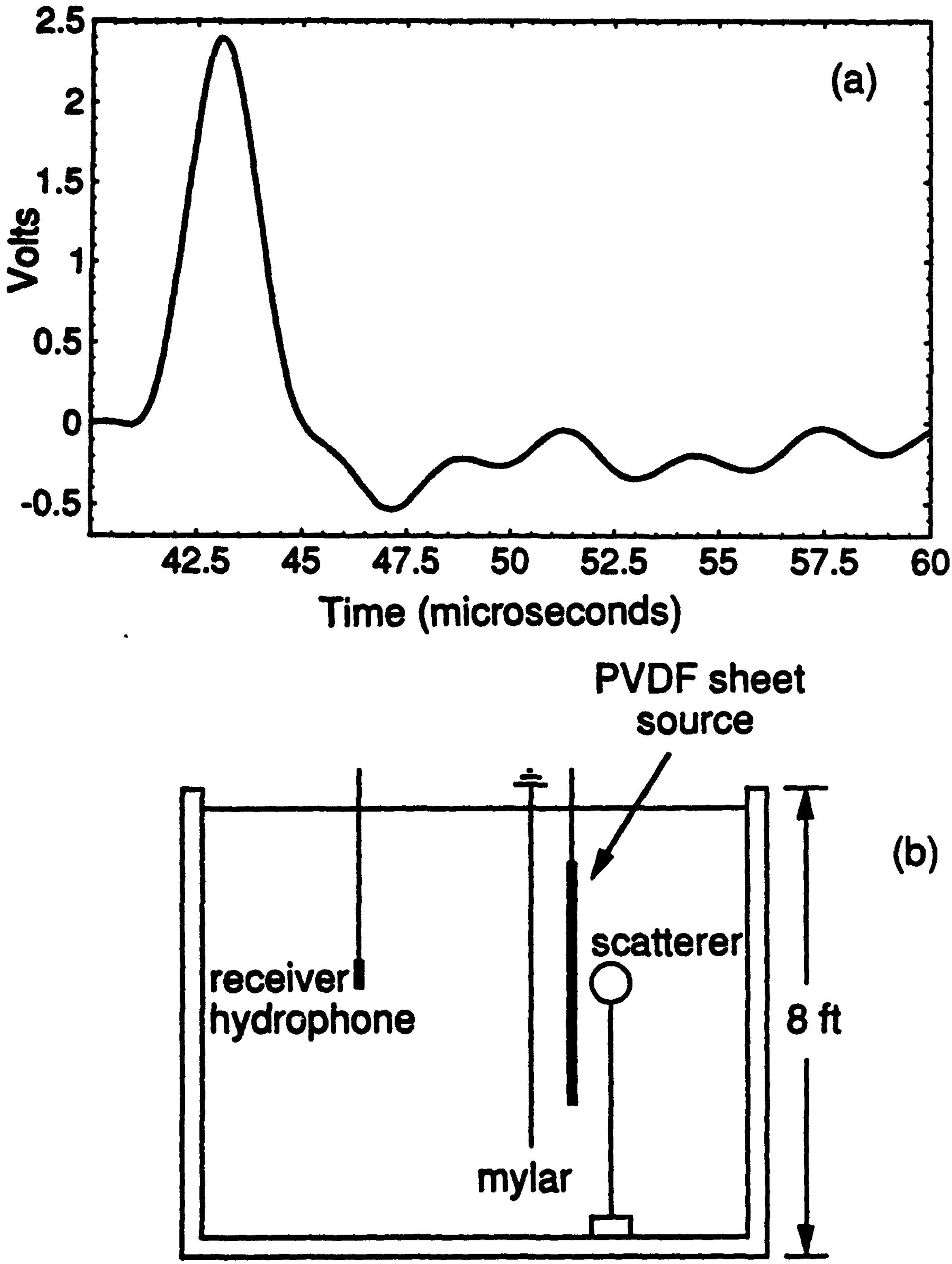


**Figure 5. (a) Impulse received by a hydrophone near the center of the PVDF sheet. (b) Configuration for backscattering record shown in Fig. 6(a).**

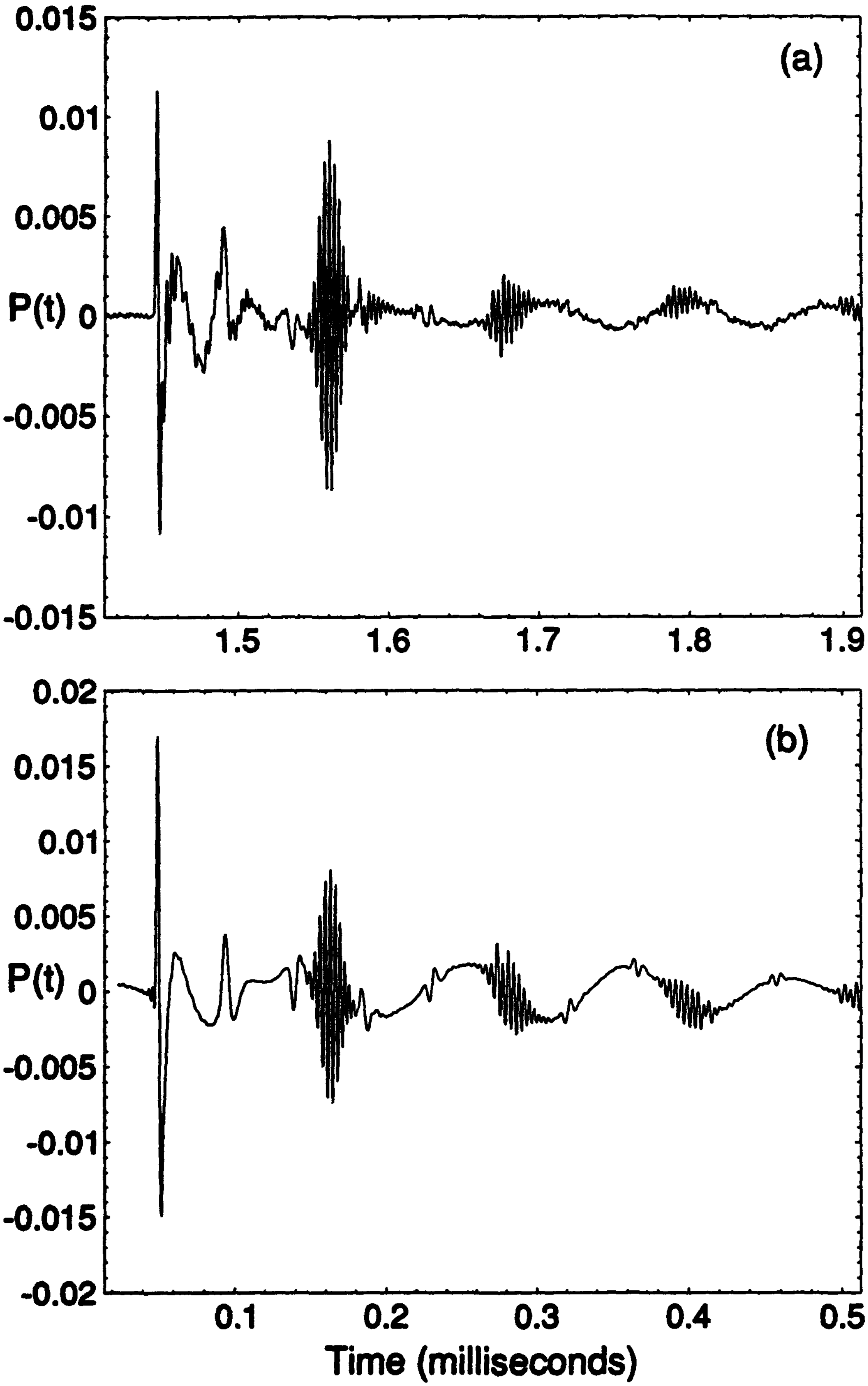


Figure 6. (a) Measured and (b) calculated impulse response of thin spherical shell.

single type of source pulse yields scattering data in the 8 to 400 kHz range. The existence of the low-frequency enhanced coupling to the $a_{0-}$ wave was evident from Watson-transform pole loci given by Hughes.[T1] It is sometimes referred to as the Junger wave [Gaunaurd and Werby, J. Acoust. Soc. Am. **90**, 2536-2550 (1991)].

### 5. Dependence of the bipolar specular reflection on the mass-per-area of a shell.

An analytical model was developed[J19] for the bipolar specular feature evident in Fig. 6 (a) and (b). **Figure 7** shows a comparison of the analytical model (dashed) with the impulse response calculated directly by a numerical Fourier transform of the form function for the shell considered in Fig. 5. The analytical theory has the following simple form

$$P(T) = \delta(T) - 2x_N e^{-x_N T}\theta(T), \quad T = tc/a,$$

$$x_N = \rho a/\rho_E h = \text{null frequency}, \quad \rho_E h = \text{mass/area of the shell of thickness,}$$

where $\delta$ is a delta function and $\theta$ is a unit-step function. The second term of P(T) gives the dashed curve which resembles the numerically calculated solid curve. This comparison (and others not shown) indicate that either the magnitude or relaxation time of the negative feature can be used to estimate $\rho_E h$.

### 6. Fresnel width of the coupling region for leaky waves on smooth elastic scatterers and radiators.

Late in 1992, Marston carried out an analysis of the angular width of the region where a leaky wave couples onto or detaches from a smooth scatterer. The understanding of this width is important to the understanding of the resolution and limitations of ray theory of Lamb waves on shells and Rayleigh waves on smooth curved solids. This work was presented and has been accepted for publication in the *Journal of the Acoustical Society of America*. **Figure 8** gives the abstract of that paper which notes a connection to radiation problems as well as to scattering. The emphasis of the paper was influenced by the experience with the Advanced Sonar Division (ARL:University of Texas) in Spring 1993.

### 7. Convolution formulation of leaky wave contributions to scattering by plates and by cylindrical shells of variable curvature.

While Marston was visiting ARL (University of Texas) he initiated development of a formulation for leaky wave contributions to scattering that applies to objects of variable

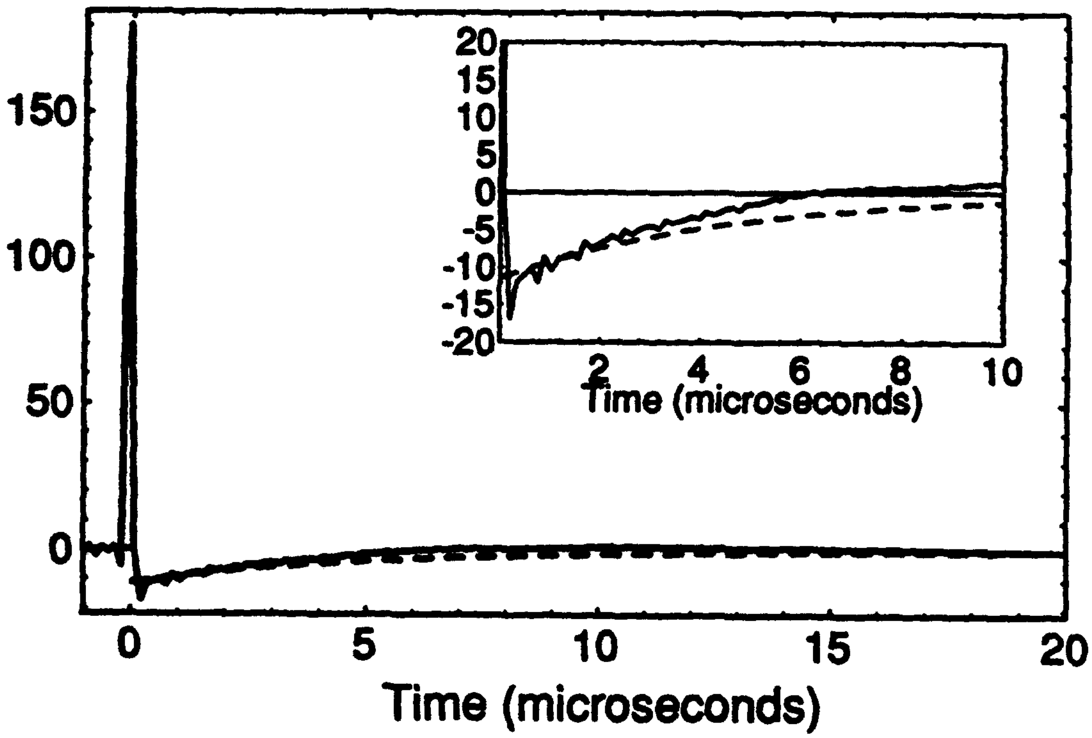


Figure 7. Comparison of theoretical results for the band-limited early-time impulse response of the 2.3% thick spherical shell considered in Fig. 6. The solid curve is based on an FFT of the theoretical form function low-pass filtered so as to suppress the fine structure associated with reverberations across the width of the shell. The dashed curve is an approximate analytical model which manifests how the magnitude and decay rate of the negative portion depends on the mass-per-area of the shell.

# Leaky waves on curved scatterers: I. Fresnel width of coupling regions and elliptical Fresnel patches

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A common procedure for estimating the apparent width of a ray in volume propagation is to construct the locus of defective paths having a phase shift that deviates from that of the true ray by $\pi$ radians [Yu A. Kravtsov and Yu I. Orlov, Sov. Phys. Uspekhi **23**, 750-760 (1980); P. L. Marston, Physical Acoustics **21**, 1-234 (1992)]. An analogous construction is described for the effective surface region for the high-frequency coupling of sound with a leaky wave on a curved surface along a given ray. Results are shown for right circular cylinders and spheres. For defective paths in the plane of incidence of the ray, the resulting Fresnel angular half-width of the patch is approximately $[2\pi/(ka \cos\theta_l)]^{1/2}$ radians where $a$ is the radius of the surface, $\sin\theta_l = c/c_l$ and $c_l$ is the leaky wave phase velocity. The analysis is relevant to estimating the degree to which ray coupling processes can be considered to be local processes. It is also relevant to the anticipation of the transition to creeping wave behavior when $c_l$ becomes sufficiently small for the Fresnel region to touch the edge of the scatterer. Defective paths outside the plane of incidence are also considered for a right circular cylinder. The Fresnel patch associated with a leaky ray to (or from) a specified point on the cylinder is approximately elliptical and the analysis is relevant to radiation as well as to scattering.

PACS Nos: 43.30.Fn, 42.20.Dk, 43.20.Tb

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Figure 8

curvature. The formulation illustrates the importance of the Fresnel width concept noted in Sec. 6 and applications have been explored subsequently by Marston with support from this grant. The work has been presented and the abstract is given in Fig. 9.[M14] The amplitude of the radiated leaky wave contribution at a surface point is parameterized by an arc length s along the surface. It is related to the incident wave amplitude $p_{inc}(s')$ at s' by the following convolution with the one-sided spatial response function h for two-dimensional scatterers

$$p_l(s) = \int_{-\infty}^{s} p_{inc}(s')h(s - s')\, ds'.$$

As noted in the abstract Marston's previous results for the coupling coefficient $G_l$ for a circular cylinder is recovered as a special case. For thin shells when the frequency is not large the phase of h is altered from the limiting form given in Fig. 9. The result may prove to be useful for the synthesis of wavefields resulting from a superposition of leaky wave contributions from randomly curved surfaces (see also Sec. C1.)

## 8. Variable phase coupling coefficient for leaky waves from resonance scattering theory.

An important aspect of the calculation of leaky wave contributions to the scattering amplitude is the description of the coupling of sound with the leaky wave. For the case of scattering by spheres and circular cylinders, approximations for the magnitude and high-frequency phase of the coupling coefficient $G_l$ were introduced by Marston [J. Acoust. Soc. Am. **83**, 25-37 (1988); **83** S94 (1988)]. Evidence of corrections to the phase of $G_l$ as $ka$ is lowered have been noted [S. Kargl, Ph.D. dissertation WSU (1990); P. Marston, *Physical Acoustics* **21**, 1-234 (1992), Sec. 5.1]. An important correction arises because the phase of $G_l$ in the original derivation depends on the phase choice for the background which may be variable in the case of shells. This low-frequency correction has been approximated by Marston in a way that should be applicable to thick or thin shells and give insight into ray theory for other high frequency scattering situations where the residues of leaky wave poles may be difficult to evaluate analytically. The result has been accepted for publication[J17] and the abstract is shown in **Fig. 10**. For the limiting case of a thin shell one of the phase corrections, which varies as $(ka)^{-1}$, agrees with a result of Rebinsky and Norris (to be published) based on thin shell theory. For thin shells, approximations of leaky wave residues have also been introduced by Ho [J. Acoust. Soc. Am. **94**, 2936-2946 (1993)], however, the derivation is not directly applicable to the cases of interest here.

**4pSA1. Convolution formulation of leaky wave contributions to scattering by plates and by cylinders and shells of variable curvature.**
P. L. Marston (Appl. Res. Lab., Univ. of Texas, Austin, TX 78713-8029 and Phys. Dept., Washington State Univ., Pullman, WA 99164-2814)

A novel high-frequency formulation is investigated that approximates the leaky wave amplitude at the scatterer in terms of a spatial convolution of the local incident wave pressure and a one-sided line response function $h(s>0)=-j\alpha\exp(-\alpha s+ik_l s)$. Here, $s$ is the propagation distance along the flat or curved surface, $\alpha$ is the reciprocal of the attenuation length, $k_l$ the real part of the wave number, and $j=1$ for equal fluid loading on both sides of a plate but $j=2$ for one-sided fluid loading of a shell or for Rayleigh waves on a solid. Application to plane waves incident on cylindrical surfaces (empty shell or solid) of slowly varying curvature yields the following far-field amplitude from a leaky ray propagating a distance S on the surface: $p_l=-2\alpha p_{\mathrm{inc}}(2\pi a_1 a_2/kr)^{1/2}\exp(-\alpha S+i\pi/4+i\eta)$, where $a_1$ and $a_2$ are the radii of curvature at the launching and detachment regions and $\eta$ is a geometrical phase accumulation. When $a_1=a_2$, the coupling coefficient $G_l$ for a circular cylinder derived previously is recovered. The result can be modified to situations where $\alpha$ varies weakly with curvature. [Work supported by ARL:UTIR&D Program and by ONR.]

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Figure 9

(Submitted to special issue of *Wave Motion* on Mid-to-High Frequency Acoustics Scattering)

Accepted for Publication

**Variable Phase Coupling Coefficient for Leaky Waves on Spheres and Cylinders from Resonance Scattering Theory**

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**Abstract**

The coupling coefficient $G_l$ is approximated for the description of the launching and detachment of leaky guided waves on spheres and circular cylinders. The approach is based on a comparison of ray and resonance scattering theory formulations which incudes effects on the phase of $G_l$ of higher order Debye approximations of Hankel functions and derivatives. The comparison also allows for a transition in the RST background between soft and rigid behavior as discussed for spherical shells by Gaunaurd and Werby [*J. Acoust. Soc. Am.* *90*,2536-2550 (1991)]. The expression for how $|G_l|$ depends on leaky wave parameters and the high-frequency limit of the phase for a rigid background are as previously described. The formulation is not restricted to thin elastic shells, however, the radiation damping is assumed to be weak. Though the phase of $G_l$ depends on a separately determined function that describes the transition in RST background, the formulation gives insight into the dependence of $G_l$ on leaky wave parameters for situations where the residues from the Watson transformation are difficult to analyze. When the thickness-to-radius ratio h/a is small, an O(1/kh) correction to the phase found by others is recovered.

Short Title: Variable Phase Coupling Coefficient for Leaky Waves

Figure 10

### 9. Retro-reflective backscattering of sound due to Rayleigh waves on a solid rectangular parallelepiped.

Consider a solid object cut with square corners which may have a random orientation relative to the direction of incident sound from a high-frequency sonar. The question of interest is to identify the most likely mechanism for producing a strongly backscattered signal. Research carried out by K. Gipson has confirmed the existence of a mechanism that is more likely to occur than specular reflection back towards the source. (The specular mechanism requires that two of the Euler angles of the scatterer lie in a narrow range while the mechanism studied here puts narrow limits on only one of the Euler angles.) The novel mechanism is illustrated in **Fig. 11**: a Rayleigh wave is launched on the stainless-steel block in water such that after reflection from two edges the radiated wavefront is directed back towards the source. It is necessary only that the angle of incidence lie within about a degree of the Rayleigh angle $\theta_R$ of the elastic material. The backscattered amplitude was measured as a function of the angle of incidence and normalized to the magnitude of the specular reflection from a solid sphere. (The solid sphere echo served as a calibration standard.) The solid curve in **Fig. 12** shows the resulting measured amplitude as a function of the angle of incidence. The dotted curve is an approximate theoretical model we have developed. While there are no adjustable scaling parameters in the model, the theoretical $\theta_R$ was offset by 0.9° in the comparison. The other important parameters are the wave number in water k and the attenuation distance $L_R \approx 2.9$ cm of the Rayleigh wave on the block. The form of the theory is simplified for $L_R \ll$ the width of the block. The experiments confirm a novel mechanism for the acoustic detection of solid objects with corners.

Marston proposed mechanisms like the one shown in Fig. 11 as a way of detecting plates with corners with high frequency sonars. This was during his visit to ARL (University of Texas) during spring of 1993. The existence of this mechanism for plates was confirmed in experiments carried out at ARL [Dodd, Loeffler, and Marston, J. Acoust. Soc. Am. **94**, 1765 (1993); Marston, Dodd and Loeffler, *ibid.*, 1861 (1993)].

## B. Radiation mechanisms for fluid-loaded membranes and plates

Matula completed a Ph.D. dissertation[T2] that describes a series of experiments on this topic with waves excited by EMATs (electromagnetics acoustic transducers). An important result of the research is the characterization of the transition radiation that occurs when a subsonic flexural wavepacket on the plate first crosses the free surface of the water. This is summarized in the abstract[M12] here in **Fig. 13** and in one section of Appendix A

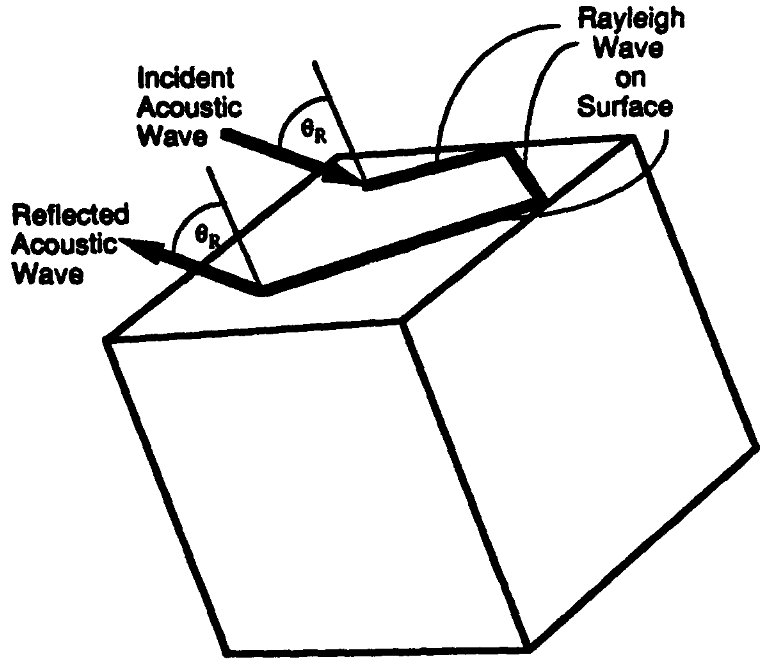


Figure 11. Ray diagram of backscattering enhancement for an elastic solid with square corners.

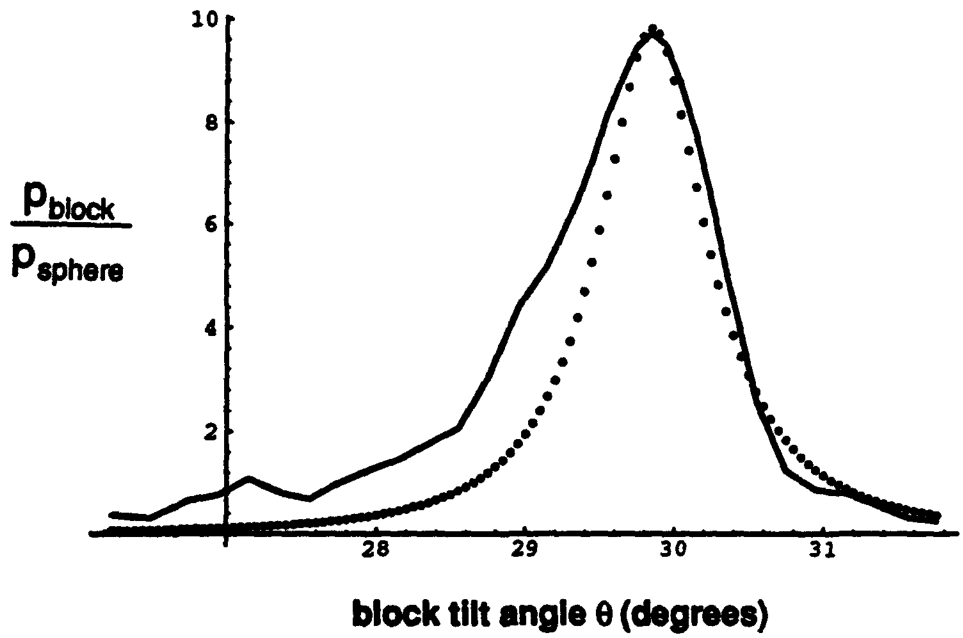


Figure 12. Plot of normalized backscattering at 1.5 MHz as a function of the tilt angle θ of the steel block which is maximized near the Rayleigh angle $\theta_R$. The points are an approximate theory with a small offset in θ as the only adjustable parameter.

**5aSA1. Energy branching of a subsonic flexural wave on a plate at an air–water interface: Transition radiation and the acoustic wave field in water.** Thomas J. Matula and Philip L. Marston (Dept. of Phys., Washington State Univ., Pullman, WA 99164-2814)

The diffraction of subsonic flexural plate waves due to a discontinuity in fluid-loading is experimentally investigated. A tone burst of flexural waves propagates down a plate, the lower section of which is submerged in water. Observations indicate that there occurs a branching of energy as the flexural wave passes through the air–water interface. A portion of the energy continues along the plate as a subsonic flexural wave with an associated evanescent wave. A second acoustic wave (which is termed transition radiation) originates at or near where the plate crosses the interface, and propagates in water to the far field. In the near field of the interface there exists an interference between the two acoustic waves in water that results in a series of pressure nulls. The pressure nulls are associated with a $\pi$ phase change in the wave field and are indicators of wave front dislocations [P. L. Marston, "Geometrical and Catastrophe Optics Methods in Scattering," *Physical Acoustics* (Academic, New York, 1992), Vol. 21, pp. 1–234]. A computation of the wave field in an unbounded fluid due to a line-moment excitation of a plate is comparable with the null pattern observed but differs in certain details. [Work supported by ONR.]

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Figure 13

of the present report. A description of the nearfield properties of the transition radiation has been submitted to the *Journal of the Acoustical Society of America*.[J18] The understanding of transition radiation is relevant to the general description of radiation by subsonic flexural waves on plates or shells in response to a sudden change in fluid loading.

## C. Caustic wavefields and diffraction catastrophes

### 1. High frequency forward scattering from a corrugated surface: Experiments.

When high-frequency sound is reflected from randomly corrugated surfaces, the reflected wavefield contains a region of large intensity fluctuations dominated by the presence of a network of random caustics. The effect of these caustics on the statistical properties of the wavefield is the subject of an investigation being carried out jointly with Dr. Kevin L. Williams of the Applied Physics Laboratory of the University of Washington. Experimental investigations are being carried out in the water tank and acoustics measurement facility developed with the support of the present grant and its predecessor[T3] (N00014-89-J-3088). [During the period of the present report, the support for the WSU graduate student on the project (J. S. Stroud) was provided by a subcontract from APL:UW from the parent ONR/ARL program on basic research.] The theoretical basis of the investigation is developed in a paper by Williams, Stroud, and Marston that has been accepted for publication in the *Journal of the Acoustical Society of America*.[J13] The reflected wavefield is expressed in terms of canonical diffraction catastrophes (Airy functions and Pearcey integrals) as well as ordinary ray contributions. The synthesis is shown to agree with numerical integration of the Kirchhoff approximation. The analysis shows how the number of contributors varies with distance from the surface such that far from the surface there are many contributors and the fluctuations become governed by gaussian statistics. Experimental investigations are in progress for both the steady-state and transient reflected wavefields. Related work of Frederickson and Marston on the properties of transient reflections near a three-dimensional transverse-cusp caustic was recently published in the *Journal of the Acoustical Society of America*.[J8]

### 2. Novel diffraction catastrophes in the scattering of light from oblate acoustically levitated drops.

Beginning with the discovery of caustic patterns in light scattered by acoustically levitated liquid drops [Marston and Trinh, Nature 312, 529-531 (1984)], observations of such patterns have been used to explore the sequence of caustic wavefields produced by changing the shape of wavefronts. The method is especially suitable for developing the analytical methods needed for problems like those mentioned in Sec. 1 (above) since the regular shape of the drop facilitates the construction of the smooth transformations needed for the application of catastrophe theory. During the period under consideration, three papers were written and accepted for publication in *Applied Optics*[J9,J10,J11] involving the study and analysis of such caustics having greater complexity than those previously considered. The abstracts of two of the papers are given in **Fig. 14.** Some of the observations involve an intense wavefield classified as an $E_6$ diffraction catastrophe that was relatively unexplored. The evolution of one manifestation of the $E_6$ studied with drop shape agrees with an analysis by J. F. Nye (Proc. Roy. Soc. London A. **438**, 397-417 (1992). Another manifestation of the $E_6$ involves a slice through wavefront parameter space that has not been analytically examined. That study is also relevant to caustics produced by perturbation of exceptionally flat wavefronts (such as those scattered by focusing spheres).

## D. Interaction of sound with sound mediated by a suspension of particles

During the previous grant, H. J. Simpson completed a Ph.D. dissertation that describes a new mechanism for mediating the interaction of sound with sound. (See the summary in the *Final Report*[T3] or the publication.[P1] **Figure 15** reproduces abstracts of the dissertation and a presentation.[M6]) The magnitude of the coherent acoustic interaction signal generated is sensitive to the initial number density of the suspension. The relaxation time for the interaction to build-up or decay following a change in the acoustic pump amplitude depends on the size of the suspended particles. A graduate student, C. Kwiatkowski, has been investigating new experimental configurations for detecting the scattered acoustic signal. The objective is to find ways of detecting the coherent acoustic interaction signal that may be easier to use than the fixed-angle large angle Bragg scattering demonstrated by Simpson. Ultimately an exploration of a wider range of suspension parameter space is planned. **Figure 16** shows some results from a modified apparatus for which the Bragg angle may be more easily adjusted. The specific test shown concerns the long term stability of the new apparatus with a Bragg angle of 41°. The magnitude of the coherent Bragg signal is plotted as a function of the probe frequency. This shows a series of Bragg peaks previously investigated by Simpson. In the new study (Fig. 16), these peaks are shown to be stable over a period of several hours when the acoustic pump amplitude is held constant. Another configuration is also being tested for the detection of the Bragg signal that is much more easily aligned.

**Hyperbolic umbilic and $E_6$ diffraction catastrophes associated with the secondary rainbow of oblate water drops: Observations with laser illumination**

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## ABSTRACT

Previous observations of oblate drops of water illuminated perpendicular to their axis of symmetry exhibit catastrophe patterns near the primary rainbow scattering angle [see e.g. H. J. Simpson and P. L. Marston, Applied Optics 30, 3468-3473 (1991)]. The present research concerns observation of diffraction catastrophes near the secondary rainbow scattering angle under similar experimental conditions. Illumination with laser light exhibits similar caustic structures in the secondary rainbow including the hyperbolic umbilic focal section and the $E_6$ or symbolic umbilic focal section. The range of drop aspect ratios explored includes aspect ratios as small as those found for freely falling drops in air as well as highly flattened drops. The new features of the secondary rainbow occur for highly flattened drops.

**$E_6$ diffraction catastrophe of the primary rainbow of oblate water drops: Observations with white light and laser illumination**

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## ABSTRACT

Oblate drops of water illuminated perpendicular to their symmetry axis exhibit catastrophe patterns near the primary rainbow scattering angle. Previous patterns include the hyperbolic umbilic focal section and separate lips events [ see e.g. H. J. Simpson and P. L. Marston, Applied Optics 30, 3468-3473 (1991)]. The present observations concern a much higher order singularity analyzed by J. F. Nye [Proc. R. Soc. Lond. A 438, 397-417 (1992)], the $E_6$ or symbolic umbilic, in the scattering by levitated drops with monochromatic and collimated white light illumination. Photographs show the colors produced when the drop is illuminated by white light. The $E_6$ occurs when the Gaussian curvature of the scattered wave front vanishes in both principal directions resulting in a high degree of directional focusing. This type of focusing, though only slightly explored, is applicable to the more general problem of scattering from penetrable spheroids.

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Figure 14. Abstracts of two of the papers accepted for publication in *Applied Optics* on diffraction catastrophes.

**4pPA9. Ultrasonic four-wave mixing mediated by a suspension of microspheres in water: Comparison between two scattering theories.** Harry J. Simpson (Naval Res. Lab., Code 7136, 4555 Overlook Ave. SW, Washington, DC 20375) and Philip L. Marston (Dept. of Phys., Washington State Univ., Pullman, WA 99164-2814)

Two ultrasonic pump waves are used to produce a grating in a suspension of 25-$\mu$m-diam latex particles. A higher frequency ultrasonic wave is used to probe the established grating to produce ultrasonic Bragg scattering. The scattering depends strongly on the pump waves and is an unusual class of nonlinearity. A previously summarized model of the interaction [H. J. Simpson and P. L. Marston, J. Acoust. Soc. Am. **90**, 2244 (A) (1991) and **91**, 2351 (A) (1992)] uses an Epstein layer approximation and effective medium approximation. This previous analysis is compared with a Fourier series approximation of the grating and the resulting predicted scattered wave fields. The Fourier series expansion is only valid when the Bragg condition is satisfied, thus giving no information off the Bragg peak. The two models match at higher pump pressures and predict similar onset pressures, but differ slightly in the medium pressure region. Both results are compared to experimentally measured scattering amplitudes for a range of probe frequencies and pump pressures. [Work supported by ONR.]

**2384** J. Acoust. Soc. Am., Vol. 93, No. 4, Pt. 2, April 1993

**Interaction of sound by novel mechanisms: Ultrasonic four-wave mixing mediated by a suspension and ultrasonic three-wave mixing at a free surface [43.25.Qp, 43.25.Lj, 43.35.Bf]**—Harry J. Simpson, *Department of Physics, Washington State University, Pullman, WA 99164-2814, August 1992 (Ph.D.).* Two mechanisms of sound interacting with sound are experimentally and theoretically investigated. Ultrasonic four-wave mixing in a dilute particle suspension, analogous to optical four-wave mixing in photorefractive materials, involves the interaction of three ultrasonic wave fields that produces a fourth scattered wave field. The experimental configuration consists of two ultrasonic (800-kHz) pump waves that are used to produce a grating in a suspension of 25-$\mu$m-diam polymer particles in salt water. The pump waves are counter-propagating, and form a standing wave field in the suspension. The less compressible particles are attracted to the pressure nodes in response to the time averaged radiation pressure. A higher frequency (2–10 MHz) ultrasonic wave field is used to probe the resulting grating. The ultrasonic Bragg scattering is then measured. The scattering depends strongly on the response to the pump wave and is an unusual class of acoustical nonlinearity. Investigation of very small amplitude gratings are done by studying the temporal response of the Bragg scattering to a sudden turn-on of a moderate amplitude pump wave field in a previously homogeneous particle suspension. The Bragg scattering has been verified experimentally and is modeled for early-time grating formations using a sinusoidal grating. The larger amplitude gratings are studied in equilibrium and are modeled using an Epstein layer approximation. Ultrasonic three-wave mixing at a free surface uses a high-amplitude 400-kHz plane wave field incident at 33° on a water–air interface to interact with a normally incident high-frequency (4.6-MHz) focused probe wave field. The 400-kHz pump wave field reflects from the surface and produces oscillating surface displacements that form a traveling phase grating.

Thesis advisor: Philip L. Marston

**1167** J. Acoust. Soc. Am., Vol. 94, No 2, Pt. 1, August 1993

Figure 15

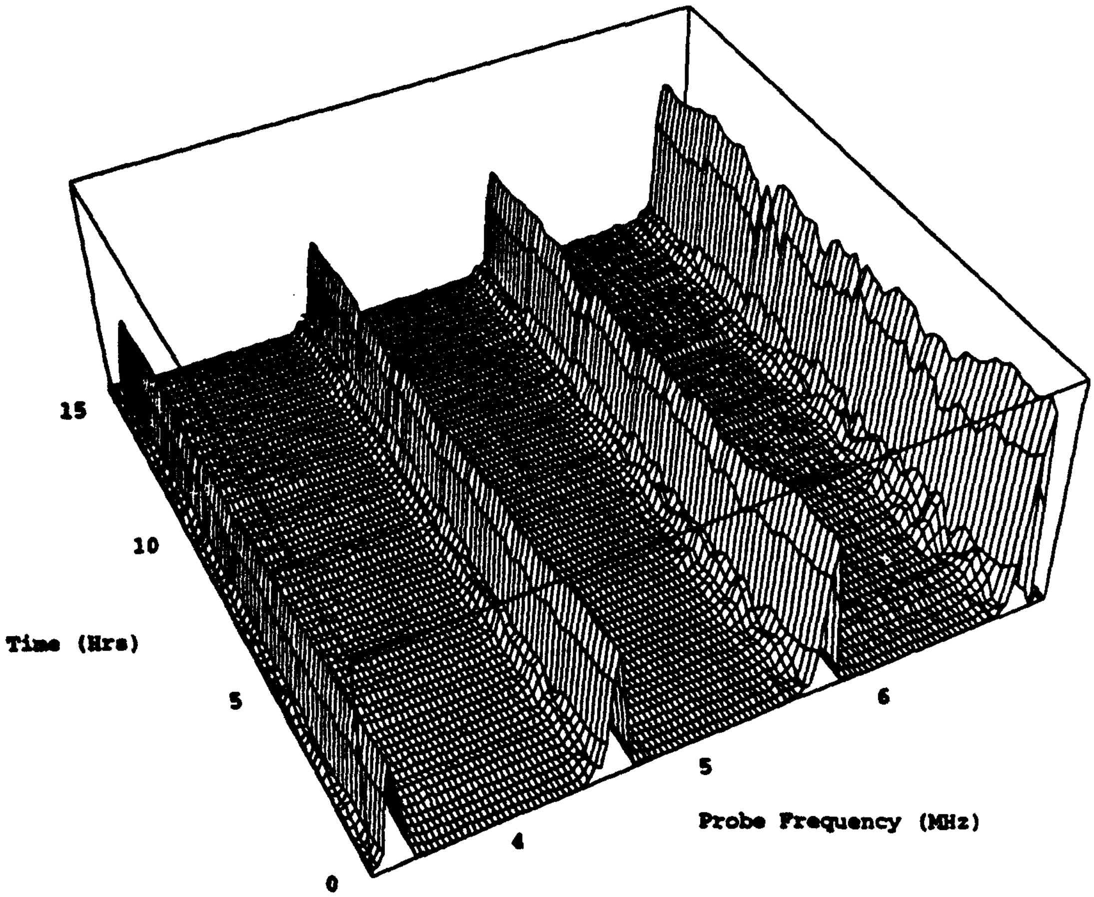
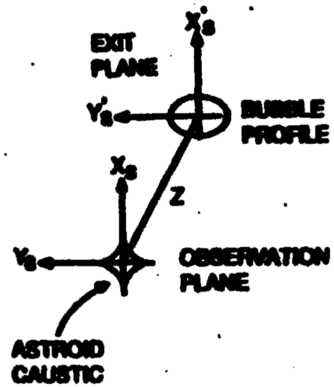



Figure 16. Long-time record of ultrasonic Bragg signature of a suspension of particles in water.

## E. Supplemental research and scholarship

Marston completed the editing of a book of *Selected Papers on Geometrical Aspects of Scattering* subsequently published by SPIE.[B1] The papers were selected from the world literature (Fig. 17) and are intended to be of value to students as well as seasoned researchers. While emphasis from the SPIE series editor was for a volume emphasizing light scattering, several papers of value to acousticians were also included. These include a translation of Debye's paper introducing what is now known as the Debye expansion. The translation was prepared with assistance from researchers that were previously students in Marston's program.[B2] In other work, an overview chapter of light scattering properties of bubbles was submitted[B3] and related previous work published.[J7] A discussion of the relationship between sound waves and Glauber coherent states of phonons was presented[M5] based on a review chapter for the *Handbook of Acoustics* prepared previously by Marston.




Selected Papers on

# Geometrical Aspects of Scattering

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***Partial contents:***

**SECTION ONE: GEOMETRICAL THEORY OF PROPAGATION AND DIFFRACTION.**

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**SECTION TWO: SPECULAR POINT THEORY, RAY TRACING, AND WAVEFRONT TRACING.**

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**Edge Diffraction Caustics and Ice Crystals**

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Figure 17

# V. APPENDIX A

Invited paper for the Proceedings of the Third International Congress on Recent Developments in Air- and Structure-Borne Sound and Vibration (M. J. Crocker editor, 1994).

## HIGH-FREQUENCY RADIATION AND SCATTERING PROCESSES FOR SHELLS AND PLATES IN WATER: BACKWARDS WAVES, COINCIDENCE ENHANCEMENTS, AND TRANSITION RADIATION

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## ABSTRACT

Coupling processes that can be important to the radiation and scattering of high-frequency sound by objects in water were investigated. One of the processes causes a prominent enhancement of backscattering by a shell as a consequence of a backward-directed leaky Lamb wave in which the phase and group velocities have opposite directions. A ray model shows how such elastic energy is strongly backscattered without propagating around the shell. The ray predictions are supported by experiments. The enhancement generally lies close to the first thickness resonance of the shell. A ray picture is also applied to enhanced backscattering of chirped tone bursts near the coincidence frequency of thin shells. The radiation by a subsonic wave on a plate resulting from a sudden change in fluid loading was also measured. A flexural wave propagating down a vertical plate in air radiates when it crosses into water.

## INTRODUCTION

Quantitative ray representations of amplitudes for the scattering of sound by hollow elastic shells in water were considered at the 1990 and 1992 Congresses [1,2]. The emphasis of that discussion was on leaky Lamb wave contributions to scattering by thick spherical shells, on representations of steady-state scattering or form functions, and on backscattering of short tone bursts near the coincidence frequency of thin spherical shells. Complete accounts of some related work have been published separately since the 1992 Congress [3-8]. The time-frequency analysis of the impulse response of shells was also discussed [2] and related discussions are given separately [9,10].

One feature that distinguishes the aforementioned line of research from related recent research by other groups [11-13] is that the usual simplification is not made that reduces the dynamical equations to those of a thin shell (as is done in Donnell's formulation of shell theory). While the resulting dispersion relations for the guided waves are too complicated to be solved analytically, the dependences of the guided wave contributions on the numerically obtained wave properties have simple high-frequency approximations [1-10]. These approximations have application at frequencies near and above the coincidence frequency and for thick shells where the usual approximations of thin shell theory do not accurately describe the wave properties.

The present paper considers other scattering processes where the full dispersion relations from elasticity theory are needed. For circular and cylindrical shells, these relations follow from the Watson transform methodology. These processes include: (i) a high-frequency enhancement of backscattering associated with a backwards wave having group and phase velocities with opposite directions; (ii) the backscattering of chirped bursts near the coincidence frequency; and (iii) the radiation from a subsonic wave on a plate due to an abrupt change in the fluid loading. Detailed discussions of these coupling and radiation processes are given separately [9,14-17], and the reader is referred to those publications for information beyond the present summary.

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Ray theory, computations, and experiments previously described [2,5-7] concern an enhancement of the backscattering by a thin spherical shell near the coincidence frequency. The enhancement was attributed to a subsonic antisymmetric guided wave designated by $l = a_{0-}$ and termed the Scholte or Stoneley wave by some authors. In the experiments and computed Fourier synthesis of the backscattering, the incident tone burst was sufficiently short that the earliest $a_{0-}$ wave contribution was later than, and distinct from, the specular contribution. Over a band of frequencies near the coincidence frequency, the earliest $a_{0-}$ contribution was larger than that of the specular echo, with a peak enhancement of approximately 3.1. Ray theory indicates that the level of the maximum enhancement depends only weakly on the shell thickness provided the thickness-to-radius ratio $h/a$ is sufficiently small that the coincidence frequency is large. (For steel the coincidence frequency is not far from $ka \approx a/h$ where $k = \omega/c$ and c is the speed of sound in water.) The experiments [6] were for a stainless steel 304 shell having $h/a = 0.0228$ with a radius $a = 38.1$ mm so that the maximum enhancement was near $ka \approx 50.8$, corresponding to a frequency of 320 kHz.

The experiment and analysis summarized below were for the same shell but with a chirped incident burst. A central question of interest is the following: Is the enhancement degraded by chirping the incident burst? It is desirable to select the chirp rate to be sufficiently rapid for the incident burst to scan through the range of the coincidence enhancement such that the total duration of the incident burst is sufficiently short for the specular echo to remain distinct from the leading $a_{0-}$ wave echo. The experiments were carried out with the shell tethered from below by a thin fishing line in a redwood tank (Fig. 1). Figure 2 illustrates the most important experimental results [14]. In (a), the burst is chirped from high-to-low as evident from the specular echo on the left which mimics the incident burst. The range swept is 218 to 436 kHz. The signature is to be compared with (b), where the incident burst is swept from low-to-high. These signatures were also compared with the signature for an incident burst of similar duration but with the steady frequency near the maximum enhancement (320 kHz). A Fourier synthesis of the signatures was also carried out for a range of sweep rates. The experimental measurements and the aforementioned comparison show the following: (i) for a downward sweep the guided-wave echo can be larger in amplitude (and narrower) than the unswept maximum signature; (ii) for an upward sweep the guided-wave echo is degraded in amplitude and generally spread out in time. The record shown in Fig. 2(a) corresponds to the sweep rate of maximum enhancement. An analysis, based in part on ray theory and properties of the impulse response, describes how the guided wave echo amplitude depends on sweep rate [14]. The rate which maximizes the amplitude tends to counteract the dispersion of the propagation time that results from the frequency dependence of the guided-wave group velocity. The guided-wave scattering process acts as a matched filter for the optimum incident burst. The experiment and analysis suggest that the thickness of the shell may be estimated by observing the backscattering of chirped bursts.

## BACKWARDS WAVE HIGH-FREQUENCY ENHANCEMENT

There has been interest and a moderate amount of conjecture concerning a prominent high-frequency enhancement that had been calculated from the partial-wave series (PWS) for backscattering by spherical shells. The enhancement has been said to be associated with a thickness quasi-resonance and resonance scattering theory (RST) methods suggest the presence of a guided wave having a group velocity with the opposite direction as the phase velocity [18]. To explain how backwards or negative-group velocity waves cause an enhanced backscattering, the usual ray diagram for leaky waves [1-4] was modified for backwards waves [19,20] as shown in Fig. 3. Furthermore, the existence of a relevant backwards wave was confirmed by Hughes [9,20,21] from Watson transform methodology based on the locus of complex ν roots of $D_\nu(ka) = 0$. (Here $D_n(ka)$ is the denominator of the nth term of the PWS for backscattering from elasticity theory and the complex index ν replaces n.) The relevant roots lie in the fourth quadrant of the complex $\lambda_l$ plane where $\lambda_l = \nu_l + (1/2)$ and the index $l$ denotes the wave type. From the locus of related roots in the frequency region of interest, the wave is identified as corresponding to the $s_{2b}$ Lamb wave on a plate where b designates the backwards wave. This identification agrees with the corresponding backward-wave root for a plate in a vacuum or in water [22]. It is compatible with the enhancement lying generally close to the resonance of a longitudinal wave across the thickness of the shell at $k_L h = \pi$ where $k_L = \omega/c_L$ and $c_L$ is the longitudinal wave speed in the shell material [2,18,23]. This corresponds to setting $ka = x_{LR} = \pi a c_L/hc$.

In our experiment a SS304 shell was suspended with a configuration similar to that shown in Fig. 1 except that a needle probe hydrophone was used to permit operation at high frequencies. For the shell used h is 2.436 mm and $a$ is 12.7 cm, giving $h/a \approx 0.0192$ and $x_{LR} = 628$. This corresponds to a frequency of 1.165 MHz, while the frequency of maximum enhancement is 1.118 MHz corresponding to ka of 601. The measured signature for a long burst incident at this $ka$ of maximum enhancement is shown in Fig. 4. The distinct specular echo is visible only at the beginning of the trace and the enhanced echo is seen to be much larger than the specular part of the echo. The backscattering enhancement in Fig. 4 is seen to be prompt since the signal builds up to its steady value more rapidly than Lamb waves are able to completely circumnavigate the back side of the shell. This is in qualitative agreement with the ray mechanism diagrammed in Fig. 3.

The diagram of the backwards-wave scattering process given in Fig. 3 leads to quantitative predictions of the contribution to the form function for backscattering [9,17,19]. The damping rate $\beta_l$ of the backwards wave is sufficiently large that it is sufficient to restrict attention to the first partial circumnavigation which is denoted by an index m = 0 as in our previous discussion of ordinary leaky waves [2-4,23]. (Backwards waves which completely circumnavigate the shell m > 0 times are too heavily damped to significantly contribute to the backscattering.) The total form function relates the farfield scattered pressure $p_s$ to the incident pressure $p_i$ in the usual way : $p_s = p_i f(a/2r) \exp[i(kr - \omega t)]$. The contribution to the total steady-state backscattering form function for the m = 0 partial circumnavigation is written [9,17,19]

$$f_l^{(b)} = -8\pi\beta_l\left(\frac{c}{c_l}\right)\exp\left(i\eta_l\right)\exp\left[-2\beta_l\theta_l\right], \quad \eta_l = -2ka\left(\theta_l\frac{c}{c_l} + \cos\theta_l\right) + \frac{\pi}{2} \qquad \text{(1a,b)}$$

where $\theta_l = \arcsin(c/c_l)$ is the launching and detachment angle diagrammed in Fig. 3 and $c_l > 0$ is the phase velocity of the backwards wave. The form of (1a) follows immediately from the generalization of the high *ka* approximation to the coupling coefficient for ordinary leaky waves [1-4,23] to the present case, while the phase $\eta_l$ takes into account the clockwise (CW) phase velocity of a wave with a counter clockwise (CCW) group velocity [17]. Equations (1a) and (1b) have also been analytically derived as a high *ka* approximation. The phase and the original motivation for Fig. 3 follow from the usual condition that the direction of the phase evolution of the surface strains is determined by the direction of the phase velocity, while the energy propagation is determined by the direction of the group velocity. The surface displacements when the ray reaches point B have a clockwise phase evolution; hence the radiation is backwards directed. The positive $\pi/2$ term in $\eta_l$ accounts for the phase adjustment from propagation of a backwards wave through the polar caustic. While the diagram in Fig. 3 shows only a CCW group velocity contribution, it is important to remember that Fig. 3 is to be rotated about the CC' axis from the axial symmetry of the shell. The expression in Eq. (1) approximates the superposition of the resulting class of ray contributions.

The approximation in Eq. (1) is generally supported by computational and experimental tests [9,17] that can only be briefly summarized in the space available here. In both kinds of tests, it is convenient to group the backwards wave contribution with the specular contribution (associated with reflection from the region around C) since as evident in Fig. 4, it is difficult to distinguish between such contributions in a quasi-steady-state measurement. The combined ray contribution becomes

$$f_{ray} = f_l^{(b)} + f_{spec}, \qquad f_{spec} = R_{pl}\exp(-i2ka) \qquad \text{(2a,b)}$$

where $R_{pl}$ denotes the unimodular reflection coefficient of a vacuum backed plate as previously described [1-4,23]. (A curvature correction [3,4,23] is omitted in the discussion which follows but has been included in a more complete discussion [17]. This omission does not affect the essential conclusions.) The computed $|f_{ray}|$ in the x = *ka* region of interest is shown in Fig. 5 as the solid curve. The evaluation of Eq. (1) makes use of the locus of zeros $\nu_l$ for the $l = s_{2b}$ wave such that $\beta_l = |\mathrm{Im}(\nu_l)|$, see [9,17]. A computational test of the ray model, $|f_{sub}|$, is shown as the dashed curve in Fig. 5. This is the residual form function contribution given by subtracting all ordinary leaky wave contributions $f_l$ from the exact complex f (from the PWS). These $f_l$ are approximated as previously described [1-4,23]. It is not necessary to subtract off any subsonic wave contributions since these are negligible in the high x region of interest. The comparison generally supports the backwards-wave approximation of $|f_{ray}|$ though $|f_{ray}|$ overestimates the combined contribution as x approaches $x_{LR}$. The important result is that $|f_l^{(b)}|$ greatly exceeds $|f_{spec}| = 1$, and the backwards wave contribution is dominant both near the peak of the dashed curve in Fig. 4 (based on the PWS) and in a plot of |f| in this region given directly by the PWS [17].

The combined specular and backwards wave contribution to f was measured as a function of *ka* from quasi-steady-state measurements like Fig. 4. Geometrical spreading factors were accounted for by a ratioing procedure with measurements taken in a different *ka* region where the specular contribution is distinct from other features. The maximum contribution to |f| agrees well with $|f_{ray}|$, though away from the peak the measurements generally lie closer to dashed curve [17]. Since the backwards wave process diagrammed in Fig. 3 does not depend on the properties of the shell in the shadow region, the approximation in Eq. (1) should apply to backscattering by a sector of a hemispherical shell. It is not necessary for the sector to be large since $\theta_l$ is typically small near the peak contribution. (For the present example $\theta_l \approx 5$ deg. near the peak.) The coupling region needs to be larger than the Fresnel width of the coupling region for the ray [24]. Notice that the mechanism diagrammed by Fig. 3 should also give enhanced backscattering by a cylindrical shell in the *ka* region of the backwards wave. We have verified the presence of such an enhancement in the PWS for backscattering by a cylindrical shell.

## RADIATION BY A SUBSONIC FLEXURAL PLATE WAVE AT A DISCONTINUITY IN FLUID LOADING

The coupling of the acoustic field with subsonic waves associated with the coincidence frequency backscattering enhancement [2,5-7,14] is a consequence of the curvature of the shell. It is well known that subsonic waves on a plate have a wavenumber lying outside of the radiation circle, so that there would be no farfield radiation from an unbounded plate [25]. Radiation to the farfield can result from scattering by objects in the evanescent region accompanying the subsonic wave [26] or from attachments to the plate, ends of the plate, or discontinuities in fluid loading. A convenient way to investigate such processes is by exciting traveling subsonic wave packets with an electromagnetic acoustic transducer (or EMAT) [15,16,20,26,27]. The results summarized here concern the radiation by a discontinuity in fluid loading for the situation diagrammed in Fig. 6. A 490 cm long aluminum plate is suspended vertically with its lower half in an 8 ft deep 3000 gal. tank of water. The plate is 12.7 cm wide and 3.175 mm thick. A flexural wave tone burst with a carrier frequency of 28 kHz is excited on the plate with an EMAT located above the air-water interface [15,16]. Tone bursts propagate away from the EMAT up the plate in air (out of the region of interest) and down the plate (toward the air-water interface). The wavelength along the plate in air is 3 cm and is determined by the coil spacing of the EMAT. The wave packet launched by the EMAT resembles wave packets launched by an EMAT on a membrane [26,27]: the displacements at the carrier frequency build up linearly in time to a steady level which is followed by a decay that is symmetric with the build up. The duration of the steady level (typically in excess of 8 cycles) is sufficiently long in the experiment to be summarized that the measurements of interest simulated steady-state conditions for the radiation process under consideration.

The frequency and mode of the wave driven by the EMAT are such that a major component of the transmitted wavefield in water is that of a flexural wave packet that propagates down the fluid-loaded plate with a subsonic phase velocity. There is also a component that radiates from near where the plate crosses the air-water interface that is termed transition radiation. Any sound radiated from the portion of the plate in air can be shown to be too weakly transmitted into the water to affect the amplitude measurements to be summarized. It is first appropriate to comment on the properties of the subsonic transmitted evanescent wave measured sufficiently below the surface that other contributions could be neglected. Let x denote the distance along the plate from the interface and z the distance from the plate. All measurements were made with a hydrophone located in a vertical plane containing the centerline of the plate. The x and z coordinates of the hydrophone were scanned by stepper motor driven linear positioning systems. In the first set of measurements x was increased incrementally beginning with 381 mm at a fixed small value for z. From the phase evolution of the tone burst the phase velocity was determined to be $c_l = 0.75 \pm 0.01$ mm/μs, which is close to the calculated value of 0.741 mm/μs from full elasticity theory for an unbounded plate in water. Using a cross correlation of signals with different x the group velocity of the wave packet was measured to be $c_{gl} = 1.50 \pm 0.02$ mm/μs, which is close to the value from the full elastic equations of 1.506 mm/μs. (The modifications to $c_l$ and $c_{gl}$ resulting from the finite width of the plate are anticipated to be small [16].) From the usual theory of evanescent wavefields [26], the pressure decays exponentially with the distance from the plate as

$$p = p_0 \exp(-\kappa z) \exp\left[ i\left(k_{xe}x - \omega t\right)\right] \tag{3}$$

where $k_{xe} = \omega/c_l$, and $\kappa = (\omega/c)[(c/c_l)^2 - 1]^{1/2}$ where $c = 1.48$ mm/μs is the speed of sound in water. The measured exponential decay, determined by scanning z, gave $1/\kappa = 4.7$ mm which is close to the theoretical value of 4.87 mm but differs significantly from the value of 5.03 mm obtained when classical (thin) plate theory is used in the calculation of $c_l$.

While the flexural wave packet appears to propagate down the plate without significant attenuation or radiation when x is large, the interface at $x = 0$ significantly affects the wavefield as a consequence of the sudden change in fluid loading there. To explore the wavefield a raster scan in x and z was made near the interface. Figure 7 shows the measured equi-amplitude contours of the pressure $|p|$ with an increment between contours of 2.5 dB. Here $k_{xe} = 0.237$ mm$^{-1}$ is the calculated wave number along the plate, as in Eq. (3), and $k_0 = \omega/c = 0.119$ mm$^{-1}$ is the wavenumber in water. (There has been an effective stretching of the z coordinate in Fig. 7 since the actual region scanned was 178 mm by 30 mm in x and z, respectively.) Some qualitative features of Fig. 7 may be summarized as follows: (i) for all values of z, $|p|$ decreases when x is decreased to lie close to the surface, and (ii) in the region scanned, there are three distinct locations where $|p|$ is small, corresponding to the three darkest patches. Additional measurements of the phase evolution as the hydrophone is scanned through the minima shows that they correspond to pressure nulls (or wavefront dislocations [28]) where the phase is indeterminate [15,16]. These properties may be understood as follows: (i) the decrease in $|p|$ with decreasing x is a consequence of an approximate pressure release boundary condition at $x = 0$; and (ii) the distinct nulls can be attributed to the interference of an outgoing cylindrical wave with the evanescent wavefield of the subsonic transmitted wave packet. The spacing of the nulls may be approximately calculated by assuming the apparent source of the cylindrical wave lies close to the free surface giving [15,16]

$$k_{xe}\,\Delta x \approx 2\pi\left[1 - \left(k_0/k_{xe}\right)\cos(\theta)\right]^{-1} \approx 12.4\,, \tag{4}$$

where $\cos\theta = [x/(x^2 + z^2)^{1/2}] \approx 0.99$. The agreement with the measured spacing of $k_{xe}\Delta x = 12.2$ supports the assertion that an outgoing cylindrical wave originates close to the interface. The amplitude of that wave must depend on the angle relative to the free surface due to the aforementioned pressure release condition. An examination of other two-dimension radiation problems with discontinuous change in surface conditions explains an additional feature of the wavefield. Barbone's geometrical construction [29], when applied to the present problem, indicates there is a region near the interface, bounded by a line of constant slope, where ordinarily evanescent wavefield of the subsonic plate wave can not penetrate. The line is labeled the "surface wave boundary" in Fig. 7. In this forbidden region there should be an absence of interference structure as indicated by the measurements.

The radiation carried by the diverging wave is termed transition radiation since it is at least partially analogous to the transition radiation of energetic (but locally subluminal) charged particles upon passing through a dielectric interface [30]. Hydrophone measurements confirm that the transition radiation is easily detected in the farfield corresponding to large values of x and z. In that region the wavefront is not cylindrical as a consequence of the finite width of the plate. The reader is referred to the original report for a quantitative discussion of the farfield [15]. The important conclusion is that the discontinuity in fluid loading causes farfield radiation as well as the nearfield interference structure. While the reflection of flexural waves by such an interface has been previously analyzed from the framework of classical thin plate theory [32], there appears to have been no previous evaluation or exploration of the details of the wavefield. Comparison of Fig. 7 with the known wavefield of a line force applied to an infinite plate [31] shows some important differences such as property (i) noted above.

It is appropriate to note that the transition radiation mechanism evident in Fig. 7 should also cause enhanced radiation by subsonic wave packets on shells with discontinuous fluid loading. For example, the shell may float so as to cross the free surface of the water. The flexural wave studied here for the plate corresponds to the $a_{0-}$ flexural wave previously discussed for shells [2,5-7] except for frequencies sufficiently low that the curvature of the shell becomes important [9]. When the fluid-loaded plate root for the subsonic wave under consideration is extended to frequencies beyond 100 kHz the root always remains subsonic, as is the case for the $a_{0-}$ shell root. This plate root corresponds to an antisymmetric wave subsonic root identified in the early work of Osborne and Hart [33].

## CLOSING COMMENTS AND ACKNOWLEDGMENT

In the coupling mechanisms summarized here the elastic properties of the plate or shell are important. In other recent publications, wavefields resulting from the reflection of sound by shells having more complicated shapes were analyzed and measured [4,34,35]. In those cases, however, the complexity of the wavefield resulted from a three-dimensional caustic produced by reflection. Geometrical properties of several other complicated scattered wavefields have been summarized [36] and some limitations of geometrical methods have been examined [4,24].

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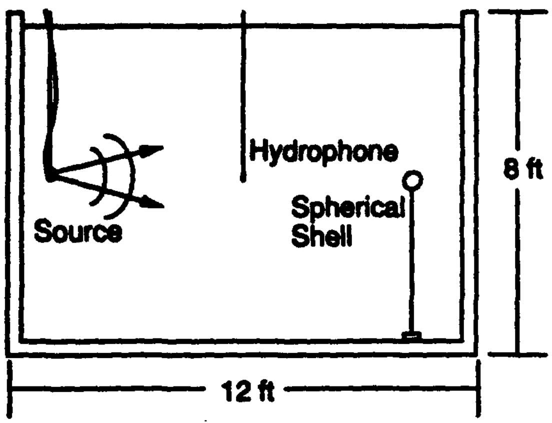


Fig. 1 -- Tank facility and source and hydrophone positions for backscattering measurements.

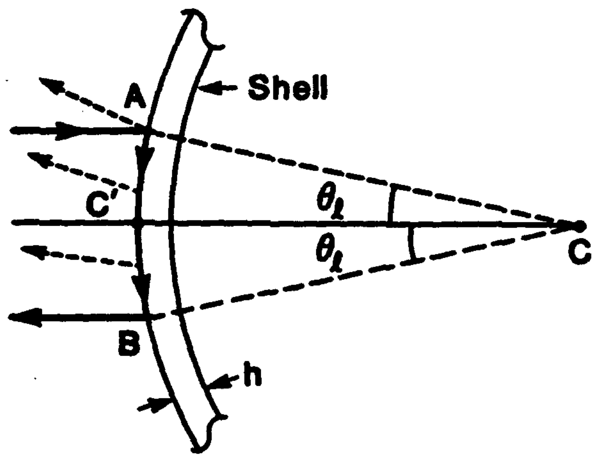


Fig. 3 -- Backwards-wave scattering mechanism for prompt enhanced backscattering in the high-frequency region where a negative group velocity wave exists [9,17,19].

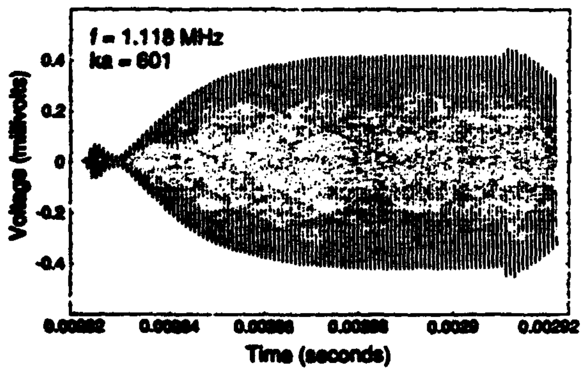


Fig. 4 -- Evolution of recorded backscattering at the frequency of maximum enhancement.

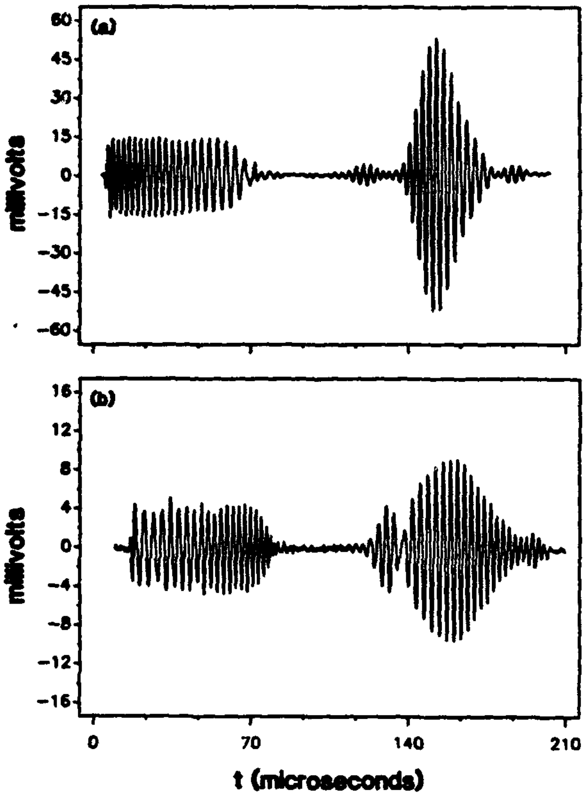


Fig. 2 -- Effect of sweep direction on the measured coincidence frequency enhancement of the earliest subsonic guided wave echo for a thin spherical shell (see text). The echo on the left is the specular reflection.

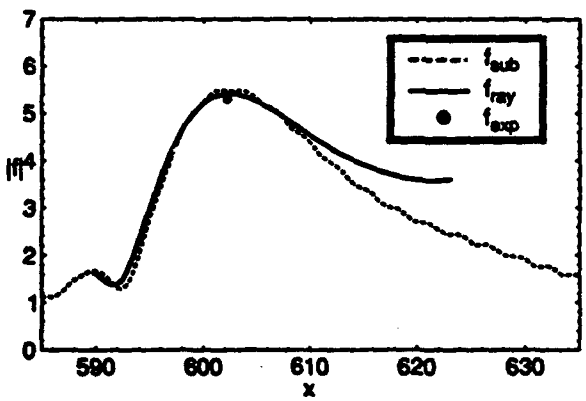


Fig. 5 -- Superposition of backwards wave and specular contributions to the form function based on Eq. (2), a modified PWS result, and experiments.

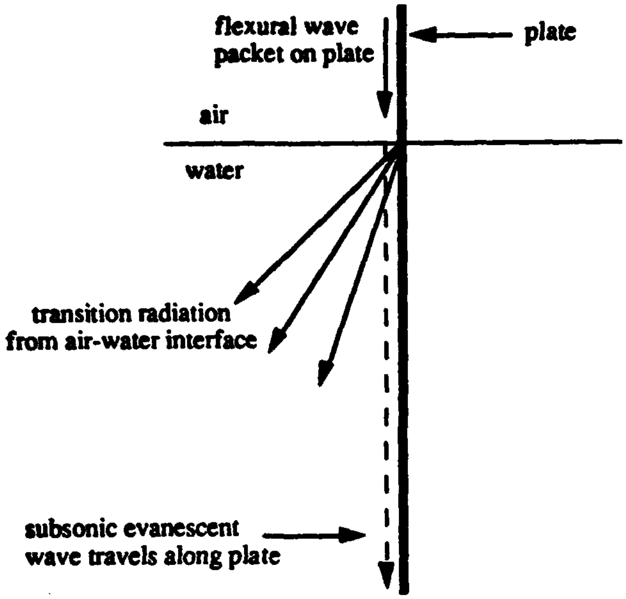


Fig. 6 -- Diagram showing contributions to the overall wavefield resulting from transmission across an air-water interface. The transmitted flexural wave is subsonic along the plate and is accompanied by a wave field that decays exponentially away from the plate. Both sides of the plate are immersed in water.

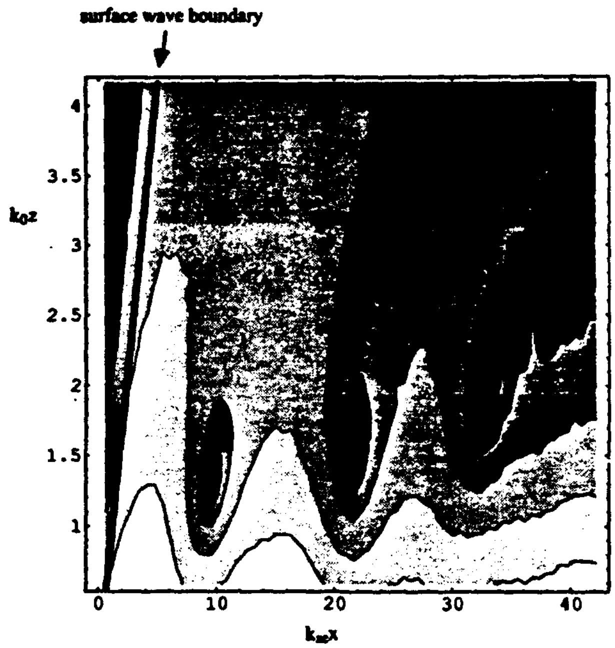


Fig. 7 -- Contour plot showing the measured nearfield pressure variations for a region close to the plate and the air-water interface [15]. The darker regions represent smaller pressure amplitudes. The distances along and from the plate are denoted by x and z.

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